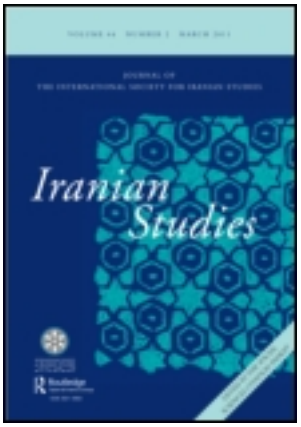

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Carol Bier

Art and *Mithāl*: Reading Geometry as Visual Commentary

This article seeks to develop an interpretation of ornament as geometric pattern that embodies metaphysical intent in Iranian monuments of the fifth/eleventh century. The proposed argument elucidates cultural meaning relevant to a particular time and specific place, with implications for broader application.[1] *Reading geometric patterns as visual commentary, this approach relates the presence of patterns in art accompanied by a Qur'anic inscription to both the practice of pattern-making and the contemporary discourse concerning mathematics, philosophy, and the Islamic sciences in Iran. Particular emphasis is placed on the use of a passage from the Qur'an (59:21–24) inscribed on the tomb towers at Kharraqān, in which the Qur'anic term,* amthāl, *is taken literally to refer to the patterns executed on the monuments.*

Introduction—Ornament as Decoration?

More than a century ago, Owen Jones isolated ornament as a subject of inquiry removed from cultural origins. His seminal work, *Grammar of Ornament*, promoted decorative schemes in architecture according to thirty-seven propositions he identified as "General Principles in the Arrangement of Form and Color."[2] During the century following publication of Jones' work on ornament, the study of geometric patterns in Islamic monuments from Spain to India has often focused on decoration and ornament through a Western lens. From this perspective, one easily and all too often infers that ornament as decoration is non-representational and, therefore, without meaning. Jones' approach had a profound impact on architectural design for many decades in the West, superseded in the twentieth century by Bauhaus and Modernist aesthetics that eschewed ornament.[3] However, the notion that ornament is devoid of

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[1]An initial attempt to present these ideas appears in Carol Bier, "Geometric Patterns and the Interpretation of Meaning: Two Monuments in Iran," in *Bridges: Mathematical Connections in Art, Music, and Science* (conference proceedings), ed. by Reza Sarhangi (Towon, MD, 2002), 67–78, with further development of a new paradigm presented at the College Art Association Annual Meetings in New York in February 2007 in a paper, "From Textiles to Algorithms: Revising an Islamic Aesthetic Paradigm." The proposed paradigm is elaborated by Carol Bier, "Number, Shape, and the Nature of Space: Thinking through Islamic Art," in *Oxford Handbook for the History of Mathematics*, ed. by Eleanor Robson and Jacqueline Stedall (Oxford, in press).

[2]Owen Jones, *Grammar of Ornament* (London, 1868 [1856]).

[3]James Trilling, *Ornament: A Modern Perspective* (Seattle and London, 2003).

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meaning—an implication Jones did not explore or articulate—has persisted to the present, in spite of the significant meanings embedded in traditional ornament in original cultural contexts.[4]

Ornament as Pattern

To consider broadly the pervasiveness of pattern in Islamic art—in virtually all media, at all times, in all places—itself suggests that pattern must be expressive of something more than just decorative intent. Logically, then, the meaning of ornament is to be found beyond the ornamental. The geometry of ornament in Islamic art, interestingly, is often a function of completion; geometry is not entirely present at the outset, but rather it becomes emergent through process. That is to say, when an artist employs a particular technology to repeat a specific design, no matter how complex the design unit that is to be repeated, it is the process of repetition that carries the artisan from initial step to the completion of a pattern.[5] Through the process of repetition, geometry emerges with the relationship of one shape to another. The pattern is not necessarily deliberative (although it may be), but rather it relies upon the interaction of a technology (such as weaving, embroidery, glazing, cutting/stacking) and a medium (such as wool, silk, ceramics, brick). The application of a technology to a medium offers a range of possibilities for executing a pattern. The processes of pattern-making in all media rely upon the interactions of craft and technology; where symmetry is the organizing principle as is the case with most Islamic monuments in Iran, the laws of nature (i.e., the limits of symmetry) underlie and structure the pattern in two dimensions,[6] a limitation empirically understood by Islamic artisans.

Taking the phenomenon of patterns as our point of departure, it becomes possible to segregate discrete aspects of patterns, acknowledging pattern-making and medium as well as geometry, cultural contexts, and meaning, allowing us to recognize divergent approaches to the study of pattern.

Approaches to the Study of Islamic Ornament in Iran

Within the study of Islamic art history in the West, several scholars have sought to understand ornament as more than just ornamental (i.e., decorative without

[4]Dorothy K. Washburn and Donald W. Crowe, *Symmetries of Culture: Theory and Practice of Plane Pattern Analysis* (Seattle, 1988) relates patterns to mathematics, and Dorothy K. Washburn and Donald W. Crowe, eds., *Symmetry Comes of Age: The Role of Pattern in Culture* (Seattle and London, 2004) explore ways that cultural information is embedded in symmetrical patterns.

[5]Carol Bier, "Choices and Constraints: Pattern Formation in Oriental Carpets," *Forma* (Journal of the Society for the Science of Form, Japan), 15/2 (2000): 127–132 [*Proceedings of the 2nd International Katachi U Symmetry Symposium* (Tsukuba, 1999), Part 3].

[6]Peter S. Stevens, *Handbook of Regular Patterns: An Introduction to Symmetry in Two Dimensions* (Cambridge, MA and London, 1981).

meaning). Richard Ettinghausen introduced the word *syntactic* to describe his approach to the study and understanding of pattern.[7] He chose to consider the whole in relation to its parts, rather than addressing individual elements. He considered the *Gestalt* as an integrated totality and he sought to determine "principal methods by which the artisans handled the extensive combinations of patterns so as to avoid bare areas which, it seems, were aesthetically unsatisfactory."[8] He defined an aesthetic "which managed to overcome in a pleasant fashion the *horror vacui*, yet did not create the impression" of overcrowding. Ettinghausen views ornament as a means for the artist or artisan to establish an aesthetic that avoided the void.

In contrast, Lisa Golombek ascribed this aesthetic to a "textile mentality" in Islamic lands, based on the extent to which patterned textiles were used and appreciated.[9] She made reference to the use of metaphors such as *hazār bāf*, or "a thousand weavings," to describe patterns of ornamental brickwork such as that of the Tomb of the Samanids in Bukhara (Figure 1). Golombek hypothesized that the intense cultural focus on textiles might account for what she called an Islamic "textile aesthetic," which seemed to inform Islamic arts and architecture in ways that distinguished it from the arts of other cultures. Acknowledging Golombek's interpretation, Oleg Grabar explored the role of ornament as a means of mediation. If one assumes that geometric principles are universal, he pondered, then what contributed to the rapid spread of geometric ornament throughout the classical Islamic world? His lecture series entitled "The Mediation of Ornament," presented at the National Gallery of Art in 1989, led to the publication of his book of the same name in 1992.[10] He surmised that there must have been particular reasons pertinent to the emergence of Islamic culture and the formation of Islamic art that fostered this extraordinarily particularistic development, which seemingly countered the universality of geometry.

Another issue raised by Grabar is equally perplexing and has generated ongoing inquiry.[11] This is the apparent absence of any substantial documentary evidence for the transmission of theoretical mathematical knowledge in its application to practice. Such theoretical knowledge, it is presumed, was held by the privileged few and would have excluded those who practiced the crafts (assuming the artisans were illiterate). At least one important work, that of Abu al-Wafā' Būzjānī, seems to admit of regular meetings among mathematicians and artisans,

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[7]Richard Ettinghausen, "The Taming of the Horror Vacui in Islamic Art," *Proceedings of the American Philosophical Society*, 123/1 (1979): 15–28.

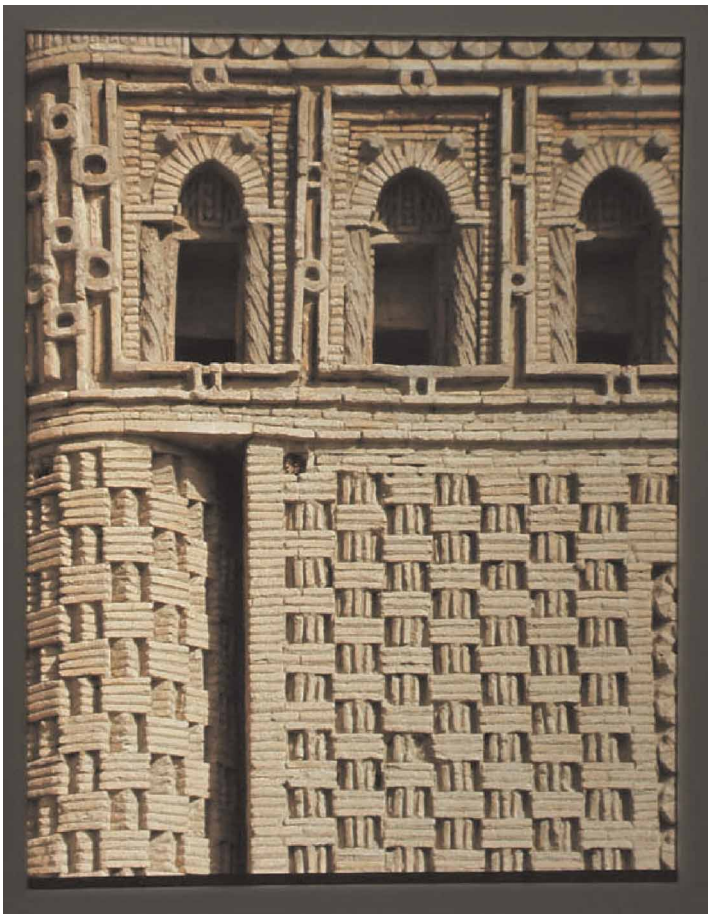
[8]Ettinghausen, "The Taming of the Horror Vacui in Islamic Art," 15.

[9]Lisa Golombek, "The Draped Universe of Islam," in *Content and Context of Visual Arts in the Islamic World*, ed. by Priscilla P. Soucek (University Park, PA and London, 1988), 25–49.

[10]Oleg Grabar, *The Mediation of Ornament*, Bollingen Series XXXV, 38 (Princeton, 1992).

[11]Marianna Shreve Simpson has assessed the impact of Grabar's provocative body of work in "Oleg Grabar, Intermediary Demons: Toward a Theory of Ornament," in *The A.W. Mellon Lectures in the Fine Arts: Fifty Years* (Washington, DC, 2002).

Figure 1. Samanid Tomb, Bukhārā, c. fourth/tenth century; exterior detail.



Source: Photographer: Hans C. Seherr-Thoss, lantern slide number E080. Freer Gallery of Art and Arthur M. Sackler Gallery Archives, Smithsonian Institution, Washington, DC.

which may have served as a key medium of transmission.[12] It now appears that aperiodic patterns with five-fold symmetry related to that of quasicrystals may have been understood by craftsmen in Iran, if not mathematicians, hundreds of

[12]Alpay Özdural, "Mathematics and Arts: Connections between Theory and Practice in the Medieval Islamic World," *Historia Mathematica*, 27 (2000): 171–201; see also Reza Sarhangi's article in this issue of *Iranian Studies*. For a contrarian view, see George Saliba's review of Necipoglu 1995, "Artisans and Mathematicians in Medieval Islam," *Journal of the American Oriental Society*, 119/4 (1999): 637–645, in which he proposes criticism of artists' methods by mathematicians.

years before this phenomenon was discovered in the West.[13] But possibilities for making patterns other than by geometric construction can result in visually complex patterns that do not necessarily require sophisticated mathematical knowledge.[14]

Other authors have chosen to explore the visual content of Islamic geometric pattern taking a stylistic approach to its decorative intent and meaning. Michael Rogers describes three distinct styles by which he identifies a chronological distinction in the fifth/eleventh century, which he attributes to the influence of Seljuk Turks, recently arrived in Iran and Afghanistan, having advanced from Central Asia westwards.[15] A more traditional art historical approach, relying solely on visual sources, seeks to find artistic antecedents for Islamic ornament; often citing the floor mosaics at Khirbet al-Mafjar, dated to the second/eighth century, as well as Roman mosaic pavements at sites in North Africa and eastern Mediterranean lands, which antedate the advent of Islam. An approach based on iconographic analysis is taken by Elizabeth Ettinghausen, who interprets geometric patterns as symbols of paradise and protection with apotropaic intent.[16] An iconographic approach is also favored by Eva Baer and Abbas Daneshvari.[17] For Baer, Islamic ornament is both supplementary and "self-contained." Baer posits cultural values regarding the decorative intent of ornament while striving to approach meaning through the representation of "things" such as fruits, flowers, vegetal motifs, fish, etc., without reference to relationships or to contemporary discourse or to metaphysical concerns. More recently, Sheila Blair and Jonathan Bloom have introduced the appellation *Cosmophilia*, "love of ornament," to offer a means of explanation for this phenomenon in Islamic art.[18]

Moving beyond ornament *qua* ornament, Gulru Necipoğlu and Yasser Tabbaa have each ascribed historical significance to the use of ornament, explaining the

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[13]Peter J. Lu and Paul J. Steinhardt, "Decagonal and Quasi-Crystalline Tilings in Medieval Islamic Architecture," *Science*, 315 (February 22, 2007): 1106–10; for earlier studies treating this topic, see Jay Bonner, "Three Traditions of Self-Similarity in Fourteenth and Fifteenth Century Islamic Geometric Ornament," in *Meeting Alhambra: ISAMA-Bridges Conference Proceedings*, ed. by Javier Barrallo *et al*. (Granada, 2003), 1–12, and Emil Makovicky, "800-Year Old Pentagonal Tiling from Marāgha, Iran, and the New Varieties of Aperiodic Tiling It Inspired," in *Fivefold Symmetry*, ed. by I. Hargittai (Singapore, 1992), 67–86.

[14]See for example the work of artists and students at the Maryland Institute College of Art, http://mathforum.org/geometry/rugs/resources/practicums.

[15]J. M. Rogers, "The 11th Century—A Turning point in the Architecture of the Mashriq?" in *Islamic Civilization, 950–1150*, ed. by D. S. Richards (London, 1973), 211–249.

[16]Elizabeth S. Ettinghausen, "Woven in Stone and Brick: Decorative Programs in Seljuk and Post-Seljuk Architecture and their Symbolic Value," in *Art Turc/Turkish Art. 10e Congres international d'art turc* (Geneva, 2001).

[17]Eva Baer, *Islamic Ornament* (New York, 1998); Abbas Daneshvari, *Medieval Tomb Towers of Iran: An Iconographical Study* (Lexington, KY, 1986).

[18]Sheila S. Blair and Jonathan M. Bloom, *Cosmophilia: Islamic Art from the David Collection, Copenhagen* (Boston, 2006).

presence of geometric patterns as signifiers of particular political agendas, or to mark dynastic heritage.[19] To this enumeration of art historical approaches to Islamic ornament we may add the theoretical work of Ernst Gombrich, who sought to categorize universal functions of ornament as "framing, filling, and linking."[20] But Gombrich never considered ornament a primary subject of artistic form.

Other studies of Islamic ornament are characterized by a more analytical approach to the geometry that is present. Keith Critchlow has analyzed works to reveal relationships of lines, angles and shapes.[21] His teaching laid the foundation for the work of Issam El-Said, who has proposed standardized systems of proportion, based upon the square root of two and the square root of three.[22] The approach of Critchlow and El-Said is particularly relevant to the fields of design and historic restoration, providing detailed instructions on how to construct or reconstruct specific geometric patterns at any scale. Despite the outstanding geometric analyses, these works, and others published in association with the World of Islam Festival Trust in 1976, are suffused with a spiritual dimension characterized as essentialist in the fourteenth/twentieth century.[23]

Each of these approaches offers insight into the visual content and forms of geometric patterns in Islamic art. Although different in interpretation of meaning, they may appear to be contradictory, but are not necessarily exclusive of one another. None, however, adequately explains the dramatic proliferation of geometric pattern in arts of the classical Islamic world from Spain to India by the fifth/eleventh century. Several scholars stop just short of acknowledging a direct relationship of geometric patterns to then contemporary religious ideas and ideology, philosophical discourse and the history of mathematics.

The recent work of Samer Akkach addresses this disjuncture, offering what he terms "an architectural reading of mystical ideas."[24] Akkach acknowledges the inherent challenge facing "modern subjects" seeking to understand "medieval objects," introducing the concept of a spatial sensibility in an effort to construct a new interpretive context. In approaching the study of Islamic ornament, Akkach argues that awareness of spatial ordering and spatial thinking facilitates an understanding of pre-modern architecture in relation to

[19]Gulru Necipoğlu, *The Topkapi Scroll: Geometry and Ornament in Islamic Architecture* (Santa Monica CA, 1995); Yasser Tabbaa, *The Transformation of Islamic Art during the Sunni Revival* (Seattle, 2001).

[20]E. H. Gombrich, *The Sense of Order: A Study in the Psychology of Decorative Art* (Ithaca, NY, 1984).

[21]Keith Critchlow, *Islamic Patterns* (London, 1976).

[22]Issam El-Said, *Islamic Art and Architecture: The System of Geometric Design*, ed. by Tarek El-Bouri (London, 1993). This work was published posthumously; El-Said's studies of the root five system still need to be brought to fruition.

[23]Tabbaa, *Transformation*, 5, 75; see also n.24 below.

[24]Samer Akkach, *Cosmology and Architecture in Premodern Islam: An Architectural Reading of Mystical Ideas* (Albany, NY, 2005).

philosophical and theological discussions with metaphysical concerns. Shifting focus from style and history to ontology and cosmology, he attempts to negotiate a course between the essentialism of fourteenth/twentieth century Perennialist philosophical perspectives and the community of art historians for whom particularistic historical interpretations may preclude acknowledgment of a Muslim worldview. Akkach refers to a perspective that reflects recognition of relationships among philosophy, mathematics, religion, and the arts.

Relationships among Philosophy, Mathematics, Religion, and the Arts in Iran

Relationships among philosophy, mathematics and religion had a long history before Islam, evident not only in works of classical Greek philosophy and Hellenistic mathematics, but also in the works of Neo-Platonist philosophers of later Antiquity, and in the reconstructed thought of Pythagoras and others involved in seeking to understand the nature of number, space and the universe. These thinkers of Classical and later Antiquity were also known to Muslim thinkers; they had an impact on thought in the Indian subcontinent as well as in the Middle East.[25]

The Abbasid court in Baghdad, capital of the Islamic empire from the middle of the second/eighth century, sponsored translations of the entire Greek scientific corpus,[26] with its emphasis on geometry and philosophy. The caliph's court, in addition to sponsoring translations from Greek texts, hosted resident scholars and delegations of scholars from India who shared their knowledge of arithmetic and astronomy.[27] Hindu numerals were introduced (our "Arabic numerals") with zero as a place-holder. New methods of calculation no longer relied upon finger-counting, although this knowledge did not reach Europe until centuries later, through the *Liber Abaci* of Leonardo of Pisa, also called Fibonacci, whose work served as a foundation for European developments in mathematics.[28]

Surviving Arabic texts provide evidence that discussions of the one and many, limits and boundaries, infinity, center and circumference, nature of the number one, and the signs of God, concepts that were as pertinent to mathematics as to theological concerns and many areas of theoretical

[25]R. Baine Harris, ed., *Neoplatonism and Indian Thought* (Albany, NY, 1982); see also Parviz Morewedge, ed., *Neoplatonism and Islamic Thought* (Albany, NY, 1992).

[26]Dimitri Gutas, *Greek Thought, Arabic Culture: The Graeco-Arabic Translation Movement in Baghdad and Early 'Abbasid Society (2nd–4th/8th–10th Centuries)* (New York, 1998).

[27]For overviews of mathematics at this time, see George Gheverghese Joseph, *The Crest of the Peacock: Non-European Roots of Mathematics* (London, 1991); J. L. Berggren, *Episodes in the Mathematics of Medieval Islam* (New York, 1986), and E. S. Kennedy, "The Exact Sciences in Iran under the Saljuqs and Mongols," in *The Cambridge History of Iran*, v. 5, *The Saljuq and Mongol Periods*, ed. by J. A. Boyle (Cambridge, 1968).

[28]L. E. Sigler, trans., *Fibonacci's* Liber Abaci*: A Translation into Modern English of Leonardo Pisano's* Book of Calculation (New York, 2002).

exploration.[29] In Baghdad, with court patronage from the eighth through the eleventh centuries, mathematicians and philosophers advanced collective human understanding of what was then at the frontiers of mathematical thought. To relate discourse in philosophy, religion, and mathematics to the arts in Iran is somewhat more complicated. There is no theoretical articulation of these relationships. The Islamic world has no equivalent of a Vitruvius or Alberti, or any of the treatises of the Renaissance and Enlightenment that poignantly address such concerns. In the absence of textual documentation that combines practice and theory (with the possible exception of Būzjānī's work),[30] it is extant buildings themselves that must guide us to an understanding of original artistic intent, whether through empirical experimentation or by the application of theoretical models. Let us, therefore, turn our attention from the study of ornament in Islamic art to the presence of patterns in Islamic art with specific reference to Iran.

We may regard patterns as offering visual evidence for the intersection of art and mathematics, with a cautionary reminder to distinguish today's perspective from speculation as to the perspectives pertinent to the times such patterns were crafted. In my own critique of Grabar's work, I called him to task for using the term, "Mediation of Geometry," rather than "Mediation of Pattern."[31] My more recent work has led me to consider the presence of patterns in art as indicative of algorithms, for which a mathematical understanding of algorithms is less relevant than an empirical understanding of the nature of pattern-making, which would have been well appreciated by craftsmen necessarily familiar with the interaction of materials and technologies. If this hypothesis is justified, the algorithmic aesthetic exemplified by Islamic patterns may pre-date theoretical understanding of algorithms as a mathematical concept.

Towards an Algorithmic Aesthetic

The emergence of a focus on pattern in Islamic art coincides temporally with the development of abstract ideas about numbers, arithmetic calculation, geometry, and algebra, articulated in contemporary discourse that explored notions of space and the spatial dimension. To relate art to these new ideas in mathematics, however, requires a conceptual leap from standard paradigms in which primary materials are treated from different disciplinary perspectives, drawing from

[29]See, for example, Majid Fakhry, *A Short Introduction to Islamic Philosophy, Theology and Mysticism* (Oxford, 1997); Oliver Leaman, *A Brief Introduction to Islamic Philosophy* (Malden, MA and Cambridge, 1999); S. H. Nasr, *An Introduction to Islamic Cosmological Doctrines* (Boulder, 1978).

[30]See Özdural, "Mathematics and Arts"; and Reza Sarhangi in this issue of *Iranian Studies*.

[31]Carol Bier, "Ornament and Islamic Art," review of *The Mediation of Ornament*, by Oleg Grabar (Princeton, 1992), and *Problems of Style: Foundations for a History of Ornament*, by Alois Riegl (Princeton, 1992), *Middle East Studies Association Bulletin*, 28 (1994): 28–30.

disparate sources and diverse methods. To relate art and mathematics requires a paradigm shift of interpretation from an aesthetic based on ornament, to an aesthetic based specifically on the mathematical idea of algorithms, which is not yet recognized as emergent in this period.

The word *algorithm* is credited to a Latin corruption of the name Khwārizmī (d. ca. 236/850), a mathematician and thinker who received the patronage of the caliph at the court in Baghdad in the third/ninth century. While mathematicians recognize the earlier use of algorithms in art, the concept of the term *algorithm* used to describe a process which is repeated again and again is not documented until the fourteenth/twentieth century.[32] The historical context, however, supports closer relationships among the arts and mathematics. Although mathematicians today do not credit Khwārizmī with the articulation of the phenomena of repetition or replication, visual sources and artistic evidence in Iran from the time of his active intellectual engagement in the third/ninth century suggest that such ideas were indeed current and may have played a key role in the development of patterns in Islamic art. In the next generation, Fārābī (d. ca. 339/950) distinguished practical arithmetic from theory as two distinct systems of thought in his *Enumeration of the Sciences*.[33]

Parallel to these developments in mathematical ideas were theological discussions that centered on the nature of God, and the relationships of man and the cosmos. These subjects were debated throughout the Islamic empire in intellectual circles at a time of passionate, vibrant, and sponsored explorations of ideas, which seem to have been halted by the restitution of Sunni orthodoxy in the fifth/eleventh century. According to Yasser Tabbaa, however, the Sunni revival ushered forth a dramatic transformation of Islamic art and architecture generated by the very ideas of occasionalism and cosmology.[34]

The coincidence of these developments is not simply temporal, although the timing may be debated. Rather, the contemporaneity of ideas and their visual expression is intimately linked, as evidenced by the specific and identifiable relationship of inscriptions and forms. In several buildings, recent studies of inscribed texts in relation to the monuments have provided key insights into intentionality that led to the particular forms of a building's construction and decoration; that particular excerpts from the Qur'an were selected to inform a building may guide our understanding of meaning in relation to intended audience.[35] In the case of the Taj Mahal, the proposition that the formal arrangements of garden and pool reflected the throne of God was first put forward by

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[32]Martin Davis, personal communication; see also John Stillwell, *Mathematics and Its History*, 2nd ed. (New York, 2002), 82.

[33]Fārābī, quoted by Martin Levey and Marvin Petruck in *Kushyar ibn Labban*, *Principles of Hindu Reckoning*, Kitāb fī uṣūl ḥisāb al-Hind (Madison, WI and Milwaukee, 1965), x.

[34]Tabbaa, *Transformation*.

[35]Erica Cruikshank Dodd and Shereen Khairallah, *The Image of the Word* (Beirut, 1981).

Wayne Begley.[36] For the Dome of the Rock, several studies have cited the significance of the Qur'anic passages selected to inform not only the building but the presence of the new religion of Islam in Jerusalem.[37] In both of these monuments, the textual choices articulate a nexus that connects monumental architecture to intellectual discourse. In the case of a tomb structure in Hamadan, *Gunbad-i 'Alawiyān*, Raya Shani has related the choice of Qur'anic inscriptions to a strong statement of Shiism in the late sixth/twelfth century.[38] A century earlier, the selection of Qur'anic passages on the tomb towers at Kharraqān (Figure 2), may link even more closely the forms of visual expression to contemporary ideas. Before turning to the significance of these forms, let us turn once again to the work of Khwārizmī in relation to visual art.

When Khwārizmī set forth his problems in *al-jabr wa al-muqābala* he described laying out dirhams to visualize the calculations (Figure 3a).[39] The post-Sasanian style silver dirhams of his time show a star and crescent at each of the cardinal points on the obverse. (In some cases where a ruler is represented, the crown replaces the fourth crescent and star.) If we conceptualize the layout of dirhams in a square grid, this arrangement closely resembles that of contemporary silk pattern-woven textiles. This style of luxury fabric quickly became the rage throughout the known world. Often described as a "framework of pearl roundels," a bolt of this type of silk is represented as a gift offered by the ambassadors depicted at Afrāsiyāb (Figure 4). This type is found in European church reliquaries, where it was used to wrap the relics of translated saints, and it occurs in nomadic chieftains' tombs in the Caucasus. Samples of similarly patterned silk textiles are also preserved in the treasuries collected by Emperor Shomu, today in the repositories at Nara in Japan.[40]

The geographic breadth of the spread of such patterned textiles has led to my consideration of an ontology of *pattern*, distinguishing a pattern (any pattern) from the medium of its construction.[41] In other words, the textile or brick or

[36]Wayne Begley, "The Myth of the Taj Mahal and a New Theory of Its Symbolic Meaning," *The Art Bulletin*, lxi/1 (1979): 7–37. More recently, Maria E. Subtelny contributes an interpretation of the polyvalence of symbolism in the Taj Mahal, based as she argues on Persian garden design, situated within the pragmatic needs of agriculture and the imaginal realm: *Le monde est un jardin: aspects de l'histoire culturelle de l'Iran médiéval* (Paris, 2002).

[37]Most recently, Oleg Grabar, *The Dome of the Rock* (Cambridge, 2006); see also Dodd and Khairallah, *The Image of the Word*, 19–26.

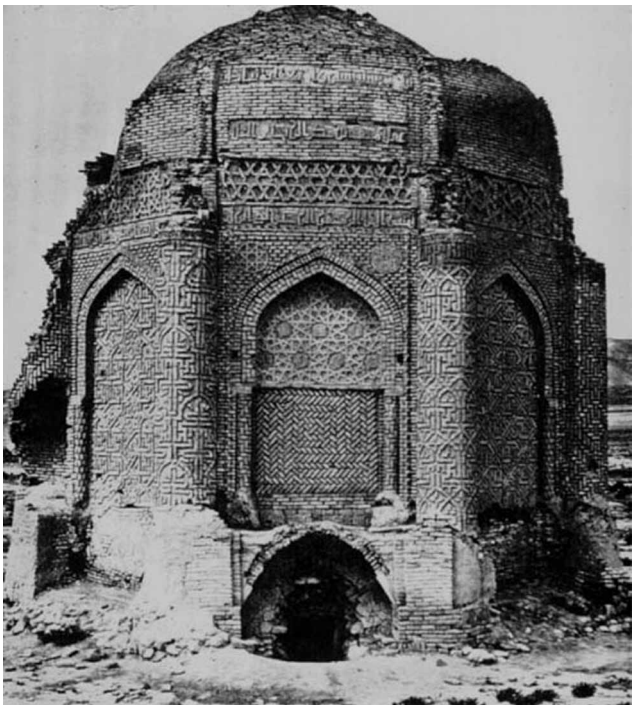
[38]Raya Shani, *A Monumental Manifestation of the Shi'ite Faith in Late Twelfth-Century Iran*: *The Case of the Gunbad-i 'Alawiyan, Hamadan*, *Oxford Studies in Islamic Art*, XI (Oxford, 1996).

[39]Khwārizmī, *The Algebra of Mohammed ben Musa*, trans. by Frederic Rosen (London, 1831). The following discussion appears in Carol Bier, "Pattern Power: Textiles and the Transmission of Mathematical Knowledge," *Appropriation, Acculturation, Transformation: Proceedings of the 9th Biennial Symposium of the Textile Society of America* (Madison, WI, 2004).

[40]See Bier, "Pattern Power."

[41]Bier, "Pattern Power"; see Carol Bier, "Patterns in Time and Space: Technologies of Transfer and the Cultural Transmission of Mathematical Knowledge across the Indian Ocean," *Ars Orientalis*, 34 (2007): 174–196.

Figure 2. Front façade. Tomb Tower, Kharraqān, dated 460H/1067–1068 CE by inscription.



Source: Front façade. Photograph courtesy, David Stronach. (Reproduced in Bier, "Geometric Patterns and the Interpretation of Meaning," fig. 3a.)

ceramic or metal, which is the substance for conveying a pattern, can be segregated conceptually from the pattern itself. The medium bears the pattern, isolated from its meaning. It is the pattern, then, that bears the content. Consider the parallel here to a book—the physicality of the book remains separate and distinct conceptually from its contents in which ideas are portrayed in words. To state this again, in the case of patterned brickwork or ceramics or a textile with a repeated pattern, it is the pattern that bears meaning, rather than the brickwork, ceramics, or textile, which is the medium carrying the pattern.

The concept of an ontology of pattern in the art is, I believe, parallel to that expressed in contemporary treatises in philosophy. Consider, for example, the distinctions made by Fārābī's regarding the genus of things and their being different from the metaphysical, and the rational principles in which he includes not only the theoretical sciences and moral and deliberative values, but also the

practical arts.[42] Through these measures the citizens may attain happiness: "To give an account of these things as they are perceived by the intellect is to give an account of their [actual] existence." The example he gives is an account of a building, distinguishing between the actuality of the building and the intentionality of the architect.[43] There is a striking parallel between the distinction of pattern from its material medium and the distinctions discussed in Islamic philosophy between "form" (*ṣūra*) and "matter" (*mādda*), actuality being dependent upon form, while potentiality resides in matter without form. Fārābī writes, "Matter . . . serves as substratum for the subsistence of form, and form cannot exist and be maintained without matter."[44]

To return to the concept of algorithms in relation to patterns, the understanding of *algorithm* as a sequence of steps, which when repeated consistently yields the same result, leads to the recognition that a pattern is created by the repetition of a unit, which is repeated according to an organizing principle. The repetition of the unit, no matter how complicated, results in a pattern that can be visually expressed in any medium. Patterns in Islamic art and architecture appear in cut brick, glazed ceramic, pattern-woven textiles, pile carpets, incised metalwork, and other materials and technologies. Let us take the simple example of an eight-pointed star. Laying one star next to the other, we may establish through an additive process a "tight-packing" of stars in which the centers align to form an underlying square grid (Figure 3b). As the stars are laid out one by one, new relationships are formed within the group; negative spaces emerge, which contribute to a dynamic visual quality. The centers of the stars all line up along axes; parallel sets of axes intersect at right angles with one another.

This unitary process that is at once systemic may be related to thinking of the *Ikhwān al-Ṣafā*, or Brethren of Purity, authors of the *Rasā'il*, or Epistles. This group of thinkers categorized numbers and focused on the number one and its successive function, relating this by analogy to the process of Creation:[45]

> The whole scheme of creation and generation resembled the generation of numbers from one: *Know, O brother, that the first thing the Creator originated and invented from the light of his Unity was a simple essence, called "The Active Intellect" just as he produced two from one by repetition. Then, he created the "Universal Celestial Soul" from the light of the intellect, just as he created three by the addition of one to two. Then he created prime matter from the movement of the Soul, just as he created four by the addition of one to three.*

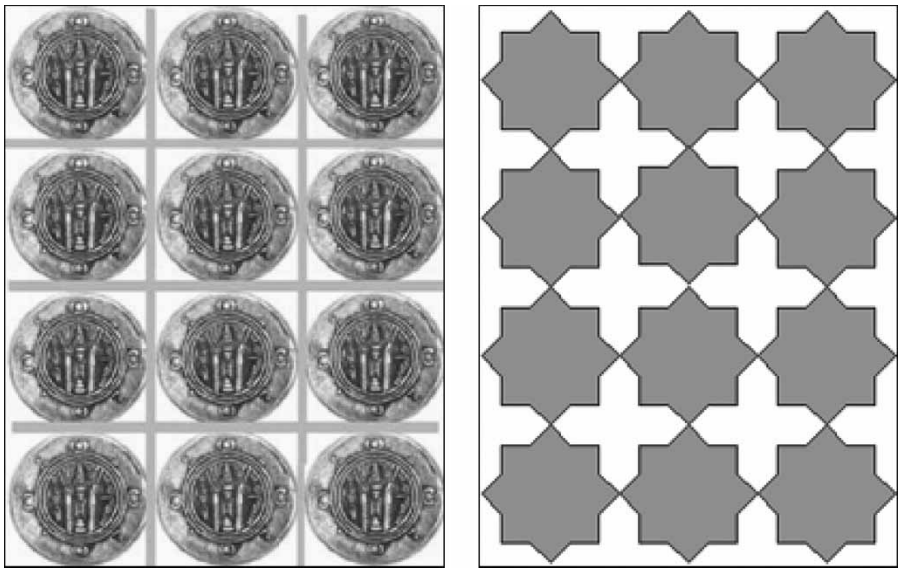
[42]Muhsin Mahdi, trans., "Alfarabi: The Attainment of Happiness," in *Medieval Political Philosophy: A Sourcebook*, ed. by Ralph Lerner and Muhsin Mahdi (Ithaca, NY, 1993), 59–82.

[43]Mahdi, "Alfarabi," 62.

[44]Al-Fārābī, *Al-Farabi on The Perfect State*: *Abu Nasr al-Farabi's Mabādī Ārā' Ahl al-Madīna al-Fāḍila*, trans. by Richard Walzer (Oxford, 1985), 109. I am indebted to Najm al-Din Yousefi for drawing my attention to this reference.

[45]Ian Richard Netton, *Muslim Neoplatonists: An Introduction to the Thought of the Brethren of Purity* (Ikhwān al-Ṣafā) (London, 1982), 34 (emphasis added).

Figure 3a. Post-Sasanian dirhams laid out to form a square grid Figure 3b. Eight-pointed stars laid out to form a square grid.



Source: Arrangements by author.

The unitary process as described leads to a systemic process of increasing complexity. The same process, described above, can be rendered visually by an additive process. The same phenomenon may be illustrated by means of a patterned textile, in which the unit is repeated mechanically by means of a drawloom (Figure 5) that relies upon a *naqqāsh*, or mock-up of the unit repeated.

In order to relate the phenomenon of ornament I would characterize as algorithmic, let us examine the tomb towers erected at Kharraqān, between Qazvīn and Hamadān. Dated by inscriptions to 460 AH/1067–1068 CE and 486 AH/1093 CE, these two monuments are elaborately encased in an architectural surface in which there are numerous geometric patterns present.[46] The earlier, eastern tower bears more than thirty patterns disposed on the eight faces of its octagonal plan. The later, western tower bears more than one hundred patterns similarly disposed, but each of the eight faces is now divided into three tiers. The patterns are constructed using cut bricks of different sizes, the bricks placed so that some project while others recede, catching the light as the sun passes

[46]For the earliest recording of these monuments, see David Stronach and T. C. Young, Jr., "Three Seljuk Tomb Towers," *Iran*, 4 (1966): 1–20. A selection of color photographs from a collection held in its entirety in the archives of the Freer Gallery of Art is published in S. P. Seherr-Thoss and H. C. Seherr-Thoss, *Design and Color in Islamic Architecture* (Washington, DC, 1968), pls. 17–26.

Figure 4. Wall-painting, Afrāsiyāb (detail). c. second/seventh century.

Source: Reproduced in Bier, "Pattern Power," fig. 1.

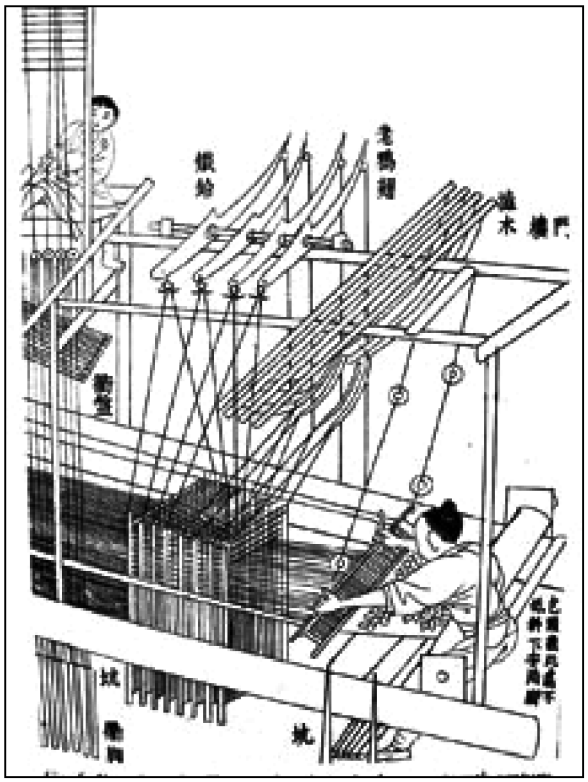
overhead revealing reflected surfaces and shadows (Figure 6). Here, the images of patterns are subjects, not representations, expressive of the natural laws that govern their construction. The geometry that is emergent depends upon the generation of the pattern, a sequence of steps repeated—an algorithm of a unit that serves as a generator for the pattern.

Geometric Patterns and Geometric Problems: Art as Mithāl

The patterns in their multiplicity are each used in a consistent manner; they organize a defined space. Each pattern exists in relation to others. The spatial divisions are such that each pattern, seemingly of infinite extent bounded only by the space in which it fits, could conceivably replace any other pattern within its defined spatial unit. Taken together, each interchangeable with one another, the patterns seem to represent something larger than the forms of their individual expression.

Each is not, then, a representation with a specific meaning, but rather pointing to something else. In this sense, the geometric patterns might be construed much

Figure 5. Drawing of a Chinese drawloom in the seventeenth century.

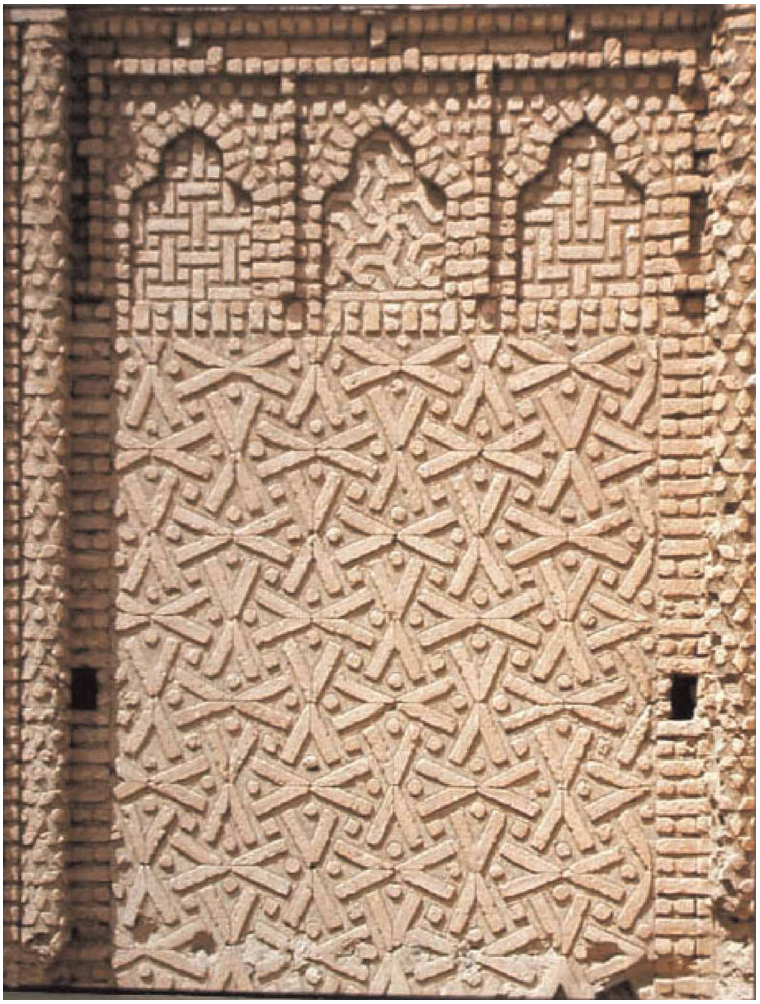


Source: Reproduced in Carol Bier, *The Persian Velvets at Rosenborg* (Copenhagen, 1995), 17, fig. 5.

in the manner of the ninety-nine names of God; no single one of them is sufficient alone, but each of them is sufficient to point to Him.

Perhaps it is not coincidental that in addition to the historical texts mentioned above, a Qur'anic inscription (59:21–24) encircles each monument, which contains numerous names of God, referring specifically to the most beautiful names (*al-asmā' al-ḥusnā)*. Although discrepant in date by more than a quarter of a century, the buildings are informed by the identical passage, executed in cut brick laid and measured carefully so as to circumscribe the entire structure, uniting all eight facets. The Qur'anic text is surmounted by a frieze consisting of a rectangular panel on each face, also patterned in brick. Consideration of the selected excerpt suggests that this passage was purposefully chosen for the eastern, earlier monument, and purposely repeated on the later, western monument.

Figure 6. Tomb Tower, Kharraqān, dated 486 AH/1093 CE by inscription; exterior detail.



Source: Photographer: Hans C. Seherr-Thoss, Lantern slide number C121. Freer Gallery of Art and Arthur M. Sackler Gallery Archives Smithsonian Institution, Washington, DC.

The first verse of the Qur'anic excerpt (59:21) contains the word *amthāl*, a plural of *mithāl*. This text is often directly associated with the Verse of Light (24:35), which also contains the word, amthāl.[47] Mithāl in Arabic has to do

[47]For discussion relating Avicenna's commentary on the Light Verse and the play of light at Kharraqān, see Bier, "Geometric Patterns and the Interpretation of Meaning," 9.

with likeness or resemblance: it may be an example, a similitude, a parable, or an allegory. The word has been used in a manner that corresponds to allegory and has been so treated in Western scholarly literature in efforts to explicate mystical aspects of Islamic philosophy.[48] An Arabic word, mithāl also came to be used in Persian texts in classical Islamic times to describe the *'ālam-e mithāl*. Corbin characterizes this as an imaginary realm, a "topography of visionary experience," an intermediary place, somewhere between the realm of the visible and the idealized, between the empirical world and that of the abstract intellect.[49] Corbin asserts that the forms and figures of this imaginal world "do not subsist as the empirical realities of the physical world . . . nor [do they] subsist in the purely intelligible world."[50] He alleges that these forms have a corporeality and a spatiality all their own, one that he characterizes as an "immaterial" materiality in comparison to the dimensionality of the sensible world. This visionary realm, which is, in effect, interdimensional, corresponds to what Corbin has called *mundus imaginalis*, "a world that is ontologically as real as the world of the senses and that of the intellect."[51]

It seems reasonable to propose that the geometric patterns at Kharraqān might be taken as literal images of the term amthāl, referred to in the Qur'anic passage that encircles each monument. If this is the case, each geometric pattern would signify more than the surface appearance of its individual artistic expression. And the range of geometric forms would visually articulate what was revealed to Muhammad and is written in the Qur'an, the artistic renderings reifying the abstract concept. Linking what is recorded visually to what is recorded verbally, we may envision that the verbal and the visual each reinforce one another, contributing to a cogent architectural program, a cycle of significant meanings and spiritual values.

This interpretation rests upon the choice of text with the word amthāl, which is usually translated as "similitudes" or "parables"—"these are the similitudes (or parables) which we offer for men to reflect" (59:21). We might then take this Qur'anic term to refer literally to the patterns, upon which men are asked to reflect. The specificity of this vocabulary may suggest that the text serves as a verbal analogue to the associated visual images of abstract patterns, helping to inform the intended meaning of these monuments at the time of their construction. If so interpreted, the patterns become visual renderings of the Qur'anic text and relate to what it is that is provided to mankind for reflection. If each geometric pattern itself serves as a mithāl, an exemplar, each offers a likeness to the other, but also a resemblance to something else.

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[48]Henri Corbin, *Mundus Imaginalis or the Imaginary and the Imaginal* (London, 1976); see also Fazlur Rahman, "Dream, Imagination, and *'Alam al-Mithal*," *Islamic Studies: Journal of the Central Institute of Islamic Research, Karachi*, 3/2 (1964): 167–180.

[49]Corbin, *Mundus Imaginalis*, 8.

[50]Corbin, *Mundus Imaginalis*, 10.

[51]Corbin, *Mundus Imaginalis*, 9.

Expanding upon this interpretation, these amthāl or patterns are placed precisely here in this context to serve as visual commentary for this Qur'anic verse.

Geometric patterns as visually expressed at Kharraqān and elsewhere in Seljuk Iran may further be considered potentially to fit descriptions of the imaginal realm, 'ālam-e mithāl or *'ālam-e khayāl*. For both conceptually and ontologically, geometric patterns may be considered as images or maps of the visionary realm—they neither map a memory, nor depict an objective reality, but are ideas generated by the mind. Expressed in this manner in brickwork, images of the mind can be seen and perceived by the senses. Once we accept the possibility that each of these patterns is a mithāl, they are categorically removed from the artistic realm of representation. Can these amthāl themselves be seen as signifiers or manifestations of the divine?

To carry this interpretation a step further, let us consider once again contemporary discourse. Fakhry defines occasionalism as "the belief in the exclusive efficacy of God, of whose direct intervention the events of nature are alleged to be the overt manifestation or 'occasion.'"[52]

Conclusion: Algorithms as Patterns, Patterns as Amthāl

No one today counters that, visually, one of the most distinct characteristics of Islamic art and architecture is the articulation of patterned surface treatments on both buildings and objects. But no one has proposed that a new theoretical understanding of the spatial dimension (two-dimensional space) is at the root of the proliferation of geometric patterns in arts of the Islamic world. Such theoretical understanding would have had to depend upon an empirical understanding of numbers and shapes, considered in relation to one another, filling the plane and limiting extent through the use of boundaries or borders. As related above, designs repeated to form patterns express algorithms. And the processes of pattern-making are dependent upon the interactions of a medium with a technology. In all cases, the processes of pattern-making, whether in ceramics, metal, wood, textiles, leather, ivory, stone, glass, or paper, rely upon the execution and repetition of an algorithm to extend a design through space to create a pattern.

The preference for patterns in Islamic textiles may surely be indicative of advances in weaving technologies (namely, use of the drawloom to reproduce designs mechanically). But such expressions seem to closely parallel developments in mathematical thinking—rendering visible the set of arithmetic operations, and functions such as halving, doubling, squaring and finding square roots, as well as algebraic forms and algorithms—that, in sum, patterns in two dimensions may be seen as visual renderings of contemporary advances in

[52]Majid Fakhry, *Islamic Occasionalism and its Critique by Averroes and Aquinas* (London, 1958), 9.

abstract thinking.[53] Theoretical understanding of the spatial dimension depended upon empirical numbers and shapes, considered in relation to space. Patterns—the repetition of a unit according to an organizing principle, such as symmetry—expressed algorithms visually. A revised aesthetic paradigm can help explain the complex web of relationships among theoretical formulations, applications, and empirical practice.

The argument presented here suggests a literal association of geometric patterns with metaphysical concerns. In particular, the interpretation of the passage excerpted from the Qur'an that informs the patterns of the two monuments at Kharraqān relates visual and verbal expression as mutually reinforcing one another. Specifically, the range and multiplicity of geometric patterns may be seen to represent the concept of mithāl, usually translated as parable or similitude. In Persian philosophy, 'ālam-e mithāl, or realm of mithāl, assumed increasing importance in the development of mysticism in the sixth/twelfth century in the depiction of visionary space.[54] Relating algorithms to patterns, and patterns to amthāl, we may perceive the patterns depicted on the two monuments at Kharraqān as an early articulation of a sacred geometry in Islamic Iran. Considering geometric patterns in the plural as amthāl, as used in the Qur'an, the tomb monuments at Kharraqān may serve to inform our understanding of a humanistic endeavor that brought to bear interrelationships among art, religion, and scientific knowledge.

When viewed from this perspective, the visual evidence offered by the rapid and dramatic proliferation of patterns in classical Islamic art may suggest the need to revise our understanding of the history of mathematical ideas to explain the derivation of the term algorithm from the name Khwārizmī. Relating geometric patterns to geometric problems, in the equation of patterns as amthāl, opens the way for a reinterpretation of both textual sources and visual sources in Islamic Iran. Critical analysis of these sources in relation to one another may foster an exploration of 'ālam al-mithāl properly situated in the spatial dimension as well as in the spiritual and philosophical realms.

[53]Roshdi Rashed situates these mathematical developments between algebra and arithmetic, and between algebra and geometry; see Roshdi Rashed, *The Development of Arabic Mathematics: Between Arithmetic and Algebra*, trans. by A. F. W. Armstrong (Dordrecht and Boston, c.1994), 3.

[54]See Corbin, *Mundus Imaginalis*.