BRIDGES 2002
Mathematical Connections
in Art, Music, and Science
pp. 67 - 78

Geometry and The Interpretation of Meaning: Two Monuments in Iran

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Abstract

The Alhambra has often served in the West as the paradigm for understanding geometric pattern in Islamic art. Constructed in Spain in the 13th century as a highly de fended palace, it is a relatively late manifestation of an Islamic fascination with geometric pattern. Numerous earlier Islamic buildings, from Spain to India, exhibit extensive geometric patterning, which substantiate a mathematical interest in the spatial dimension and its manifold potential for meaning.

This paper examines two monuments on the Iranian plateau, dating from the 11th century of our era, in which more than one hundre d exterior surface areas have received patterns executed in cut brick. Consi dering context, architectural function, and accompanying inscriptions, it is proposed that the geometric patterns carry specific meanings in their group assemblage and combine to form a programmatic cycle of meanings. Perceived as ornamental by Western standards, geometric patterns in Islamic art are often construed as decorative without underlying meanings. The evidence presented in this paper suggests a literal association of geometric pattern with metaphysical concerns. In particular, the argument rests upon an interpretation of the passages excerpted from the Qur'an that inform the patterns of these two buildings, the visual and verbal expression mutually reinforcing one another. Specifically, the range and multiplicity of geometric patterns may be seen to represent the Arabic concept of *mithal*, usually translated as parable or similitude. The Persian, *alam-e mithal*, or realm of *mithal*, assumed increasing im portance in the development of Iranian Islamic philosophy and mysticism in the 12th century in the depiction of visionary space. The arguments presented here suggest that the patterns depicted on these two monuments articulate a sacred geometry in early Islamic Iran.

1. Geometry in Islamic Art and the Paradigm of the Alhambra

Mathematical aspects of Islam ic art inform a beauty of form, pattern, and structure. Thro ughout most of Islam's history and in many regions of the world, Isla mic art and architecture have been characterized by a predilection for geom etric patterning. Co mbined with Arabic calligraphy and stylized floral ornament, geometric pattern is recognized as a key component of the visual vocabulary of Islamic art, ranging from the monumental (architecture) to arts of the object, illu mination and painting i n arts of the book. Wit hin the Islamic world, pattern m aking has served a prim ary function in t he organization of two- and thr ee-dimensional space, in architecture as well as in objects of all media. Patterns that may appear initially to be complex and intricate can be analy zed to identify a unit and it s repeat acco rding to an a lgorithm, often relying upon the principles of symmetry and exemplifying a process that is at once unitaryand systemic.

The monument most often cited to document an Is lamic fascination with pattern and its inherent geometric possibilities is the Alham bra (fig.1a), palace of the Nasrid rulers of Spain at Granada. Several authors have stated that t he seventeen plane sy mmetry groups are pres ent within its wa lls, yet no clear evidence remains of a theoretical understanding be yond that obtained through empirical experimentation or the hypothetical use of pattern books. Whether or not all seventeen field patte rns are present, what is apparent to a ny visitor is that the interior walls, va ults, and ceil ings of t he Alhambra indeed displa y an exuberant array of patterns executed in a variety of materials, including carved stucco and p laster, painted wood, carved marble, and ceramic mosaic.

The Alhambra, as the last stronghold o f Muslim rule in the Iberian peninsula, was taken o ver by Ferdinand and Isabella in 149 2, bringing to com pletion the C hristian *reconquista* of Spain. But t he significance of Islam ic geometric ornament did n ot end with Christian domination. The opulence of t he Alhambra captured the imagination of the widely published Am erican author, Washington Irvin g, as

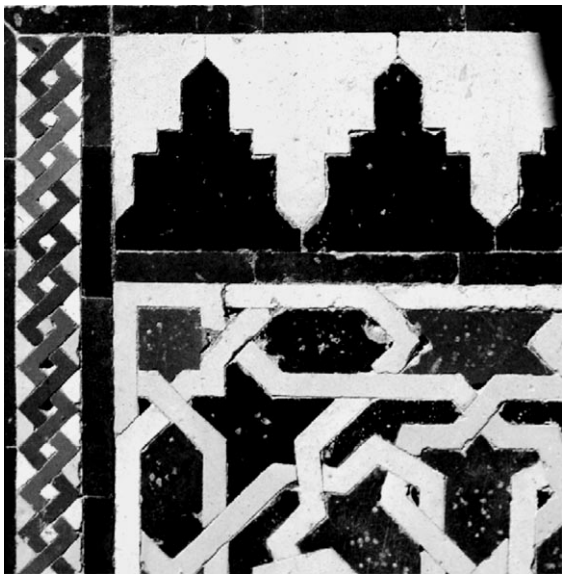

Fig. 1a Detail of geometric pattems in ceramic mosaic at Alhambra Palace, Granada Spain, 13th C.

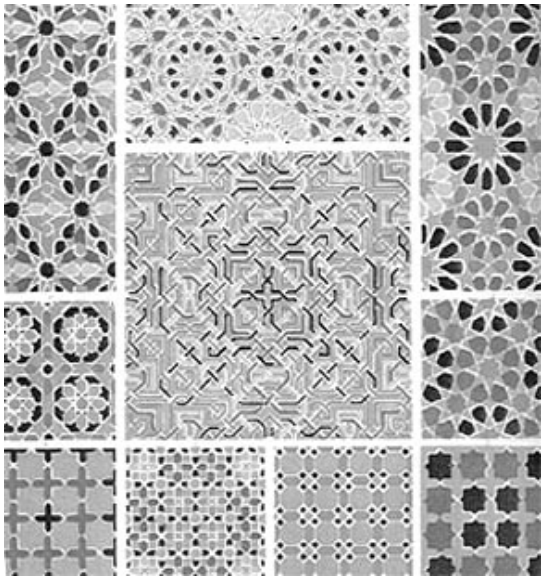

Fig. 1b Moorish ornament, published by Owen Jones, *Grammar of Ornament* (1856).

well as that of European artists, in particular those known as Orientalist painters in the 19th century. At the forefront of British design in the middle of the 19th century, Owen Jones carefully measured and documented decorative patterns at the Alhambra in relation to architecture; his meticulous publication of plans, sections, and elevations [17], contributed to the growing influence of Oriental exoticism in European decorative arts. Jones' studies of the Alhambra (fig.1b) sustained his theoretical formulations of principles of design, which he proposed were comparable to natural laws [18]. In the 20th century, M.C. Escher's mesmerizing fascination with tessellations is usually attributed to the long-term effects of his careful on-site analyses of the Alhambra's patterns, preserved in his notebooks. The impact of the Alhambra on European and American sensibilities relies upon the visual appeal of color, texture, and multiple rhythms, coupled with fantasies rife with cultural interpretations resting upon an appreciation for the opulent luxury and exoticism with which the Orient penetrated Western imagination.

2. Earlier Explorations of Geometry in Islamic Art and Culture

At the Alhambra, the profusion of patterns in ceramic, stucco, painted wood, and other media, visually demonstrates the limit of seventeen plane symmetry groups. But many earlier Islamic monuments -- from Spain, North Africa, and the Middle East, to Iran, Central Asia, and the Indian sub-continent -- also exhibit exuberant play with patterns that rely upon symmetry, intensely exploring visual repetition that expresses relationships between number (arithmetic) and space (geometry) [7]. In each instance of a symmetrical pattern (that is, a pattern in which the organizing principle is symmetry), the reiteration consists of an algorithmic iteration of a defined unit and its repeat. Yet of all the Islamic buildings that exhibit an appreciation of symmetry and pattern, with an infinite potential for artistic development, for the Western world it is the Alhambra that has remained pre-eminent.

The treatment of pattern in Islamic monuments seems to flourish particularly from the 11th century onwards, in a manner that is at once both universal within the Islamic world, and sufficiently distinct a treatment from other cultural traditions that it may be, indeed, characterized as Islamic [15]. Just what may have led to the proliferation of geometric ornament within the Islamic world, and why it proved to be both fascinating and enduring as an expression of Islamic art to the present, is not yet conclusively understood. New interpretations have recently been put forward to associate geometric pattern with dynastic lineage [22], or with the restoration of Sunni orthodoxy [33].

To judge from datable works of Islamic art and architecture, the 11th century of our era seems to have been a period of intense experimentation, one in which relationships between numbers and shapes, and

emergent geometric patterns, were explored by both artisans and builders [24; 7]. Concurrent with the active patronage and vibrant pace of production of many works of art and architecture, the 11th century also witnessed the interests of mathematicians and philosophers, who explored aspects of patterns and pattern-making in relation to time and space. One of the great mathematicians of this period was al-Khwarezmi, from whose name we derive the word algorithm, and from the title of one of his works, *Al-Jabala wa'l muqabala*, we derive the word algebra. During this period in central Islamic lands (Iraq and Iran), dramatic advances contributed to understanding algebra, the development of trigonometry, explorations of spherical geometry, and, in general, innovative applications of mathematics in the real world [19]. This was also the time of Omar Khayyam [26], whose mathematical interests have recently been linked to the building of the north dome of the Masjid-i Jomeh in Isfahan [23].

Between the 9th and 11th centuries recurring topics in mathematics and philosophy may be identified in Islamic lands, extending from Iraq and Iran to Syria, Egypt, and across North Africa, and as well as to Khurasan (Central Asia), India and Spain. Broadly conceived, these ideas concerned number, shape and the nature of space, and notions of spatial and temporal infinity. This period saw the introduction of zero and Arabic (Hindu) numerals, and the decimal system from India, the theoretical beginnings of algebra and trigonometry, advances in astronomical calculations to determine the direction of qibla for prayer and pilgrimage as well as for the orientation of religious buildings, and applications of mathematical advances in many areas of human endeavor [6]. It was also a period of religious fervor focused on finding explanations and justifications for, or rationalizations of, prophecy and revelation, and developing proofs to confirm the existence of God and the veracity of the Islamic revelation [2].

In the absence of textual documentation of a theoretical nature (the Islamic world has no equivalent to Vitruvius or Alberti), it is buildings themselves that must guide us to an understanding of original artistic intent, whether through empirical experimentation or by the application of theoretical models. Two

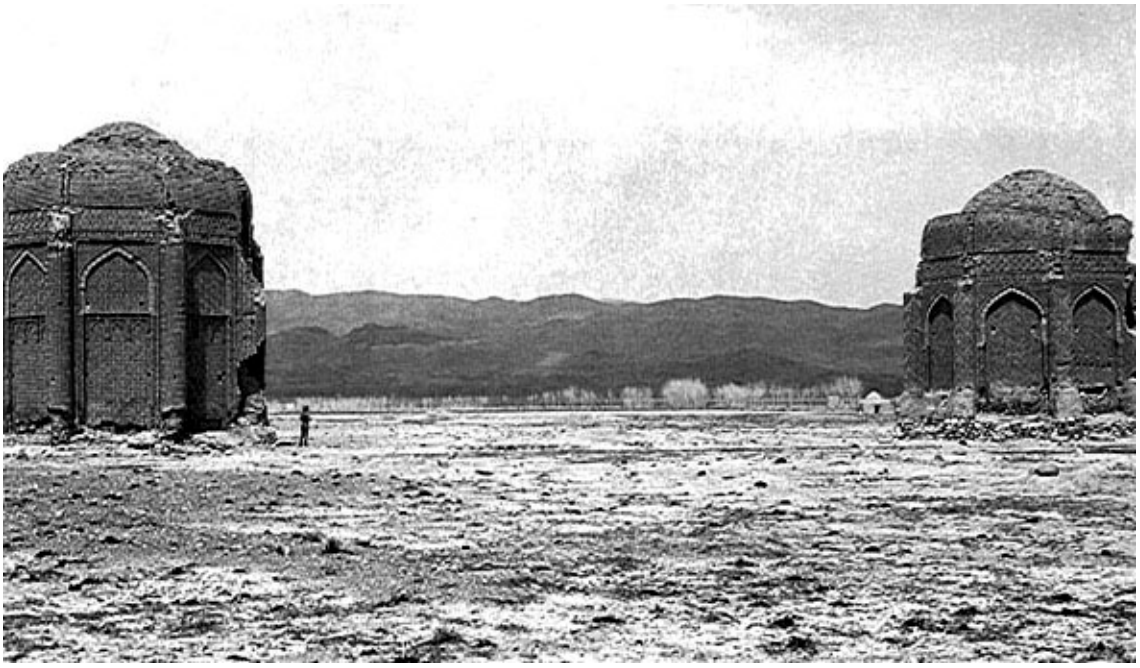

Fig. 2 Two tomb towers at Kharraqan (looking north), Iran, 11th century.
Reproduced with kind permission of David Stronach [32]

monuments constructed in the 11th century stand out as visually exemplary in their treatment of geometric pattern, but they also bear inscriptions that may lead us towards a clearer understanding of the meanings that underlie the uses of geometry in Islam at this time. They are a pair of tomb towers in western Iran (fig.2), located at Kharraqan, which lies off the main route connecting the cities of Hamadan and Qazwin; they are sufficiently remote that their existence remained unknown to Western scholars until the second half of the

twentieth century [31; 32]. These two monuments stand as testimony to an empirical treatment of algorithms, expressed visually as a unit and its repeat, to create geometric patterns.

3. The Tomb Towers at Kharraqan

3.1 Architectural Form and Decoration

The two monuments at Kharraqan today stand as a pair of octagonal towers rising more than twelve meters above the plain (fig.2). Constructed of brick and once surmounted by a dome, each tower displays eight niched rectangular vertical faces, which are joined by cylindrical engaged buttresses. The enclosed interior space is nearly eight meters in diameter. In spite of their appearance as a pair, the two monuments were constructed at different times (see below). They bear several distinguishing structural features, which are not relevant to the consideration of the treatment of their geometric patterns.

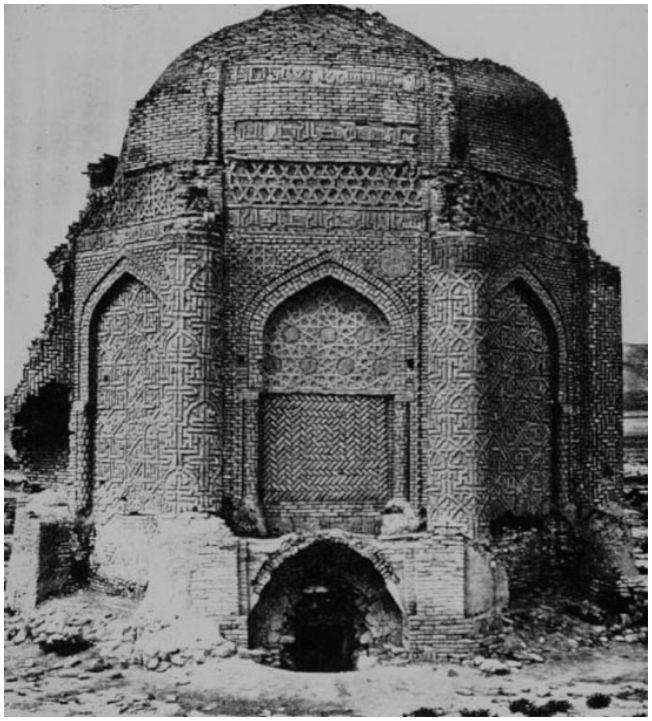

Fig. 3a Tower I (eastern), detail of facade, Kharraqan, Iran.
Reproduced with kind permission of David Stronach [32]

Fig. 3b Tower II (western), detail of facade, Kharraqan, Iran.
Photography courtesy of Ann Gunter.

What is most striking in appearance is that the exterior surfaces are entirely covered with geometric patterns, executed simply by the selective cutting and placement of bricks set with mortar. In each building, the face with the entryway (fig.3a-b) has received the most elaborate treatment. Considering the entire exterior surface, some areas show the mortar layer flush with the surface of the bricks (or nearly so); in other areas the mortar is deeply recessed, allowing the play of light and shadow to help define the patterns of bricks. The entry façades, including the entry face and the faces adjacent to it on each side, show a greater proportion of recessed mortar, giving a sense of higher relief to the patterns in contrast to those faces not visible from the front of the building (fig.2). Taken together, these monuments provide early and incontrovertible evidence for a truly passionate fascination with the phenomena of geometric patterns.

The two buildings are clad in bricks set as revetment, which comprises an extraordinary array of geometric patterns. Although not identical in form, all of the patterns are used in an identical manner, serving visually to clothe the building. In both buildings, the elaborate brick patterns define the vertical surfaces, leaving no exterior parts beneath the dome unadorned.

Despite their apparent similarities, the two buildings each have an individual character, distinguished by features that suggest meaningful intentionality extending beyond merely formal similarity. The eastern tower, which is earlier, bears more than thirty patterns disposed on its eight faces and connecting buttresses. The western tower, constructed about a quarter of a century later, has well over seventy patterns on its exterior surfaces. In order to maximize the areas appropriate to receive ornamentation by patterning, the niched facades of the later tower are subdivided to form an upper and lower section by the inclusion of a frieze of three small trilobed arches (fig.3b). Each of these arches, in spite of its small size, bears an overall repeat pattern, large in scale in comparison to the dimensions of the arch. And each of these patterns implies its extension beyond, to fill a potentially infinite expanse, contained only by the form of the arch itself. In addition, other minor areas have also received attention by way of pattern in cut bricks: the engaged columns at the sides of each arch, the soffit of the arcuate projection itself, and the spandrels above each arch.

The play of patterns, in every instance, is based upon an algorithm consisting of a unit configuration of cut bricks that is consistently repeated to fill a designated space. Geometric relationships thus revealed are emergent structures within the pattern that is at once unitary and systemic. The presence of a third dimension, created by the relationship between bricks and mortar, seems almost incidental to the two-dimensionality of the pattern as perceived by the viewer. The brick patterns are delineated by the careful cutting and placement of bricks, set upon beds of mortar that are recessed such that the daily appearance of the movement of the sun across the sky highlights the bricks, casting shadows in the interstices. The play of darkness and light articulates the visible surfaces of the buildings, appearing on all eight faces of each monument as well as on the engaged columns or buttresses, which serve both to separate and join adjacent faces. The entryway of each monument, as well as the adjacent two faces, comprising five faces together, shows recessed mortar joints, whereas the back three faces, and adjoining buttresses, show flush mortar joints. This feature, combined with other aspects having to do with the distribution of patterns, suggests a clear sense of symmetry in the conception of each building.

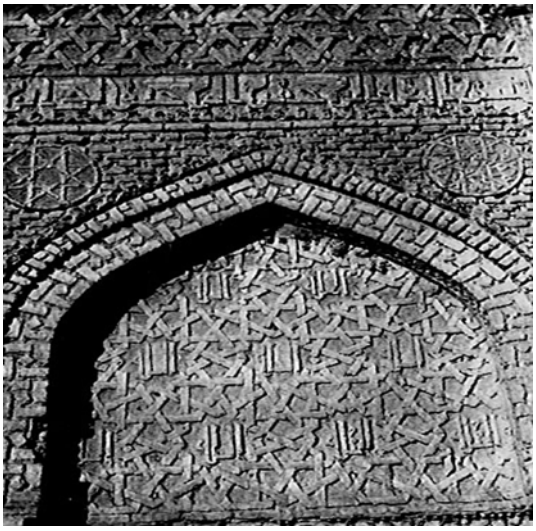

Fig. 4a Tower I (eastern), detail of tympanum above entry.

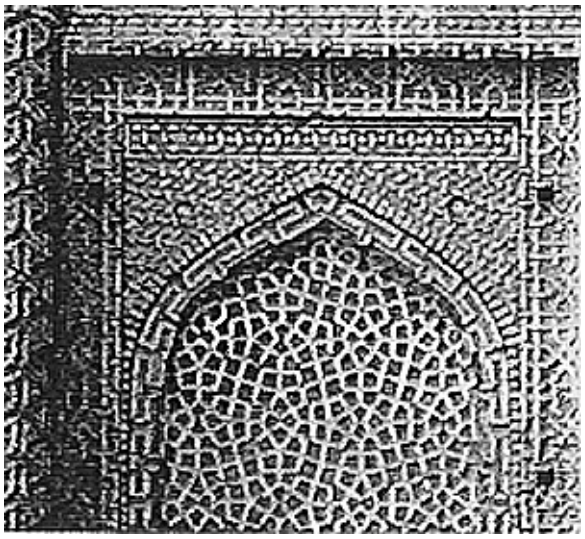

Fig. 4b Tower II (western), detail of tympanum above entry.

Reproduced with kind permission of David Stronach [32]

Further, the face in which the entry to the interior is set serves as a central focus, both with richer decoration and with the appearance of higher relief, than that of any of the other faces. In addition, the buttresses adjoining this face, and adjacent faces, have each received additional attention to patterns, in contrast with the five faces not visible from the front of each monument. In the case of the earlier tower, the tympanum above the entrance (fig.4a) bears a geometric pattern of interlace which form twelve-pointed stars in negative space; in each of these stars, occurring nine times, is the name, Allah (God). In the later tower, an unusual repeat pattern with pentagons fills the tympanum (fig.4b) above the historical inscription, which lies above a Qur'anic inscription that is not present on the earlier tower.

Aside from the fact that the entryway is reserved in each instance for the most complicated pattern, which is placed in the tympanum above the entrance, there is no sense that any one pattern is more significant than any other. As for geometry, there does not seem to be any consistency or progression in the choice of patterns. The eye is not drawn to a single central focal point. Rather each panel and area designated by pattern presents multiple centers. Sometimes, as in the case of interlaced octagons and interlaced dodecagons, the centers are specifically marked by a projecting dot. The range of patterns displays an evident awareness, even if by way of experimentation, of the play of symmetry with its inherent ambiguities and emergent geometric relationships. Horizontal and vertical reflections and glide reflections are much in evidence, as are rotational symmetries. There is clearly play with the cut-brick units, which combine to form triangles, squares, hexagons and six-pointed stars, octagons and eight-pointed stars, and dodecagons. There is considerable attention given to illusionary interlace, visually effected by the selective cutting of bricks and their specific juxtaposition. The two bosses above the entrance in the earlier, eastern tower (fig.4a) suggest a particularly expressive contrast. Set against the paradigmatic symmetry that is evident throughout the two buildings, the distinction between an octogram and a hexagram is potentially obscured by the presence in each boss of a pair of parallel vertical lines.[1]

3.2 Architectural Function

The form of these towers, both in plan and in elevation, suggests a categorical relationship with tombs of Seljuk date on the Iranian plateau [16]. This group of monuments relies upon rather strict geometry in three-dimensional space, the stark volumetric mass based upon cubes or cylindrical forms, in the latter case, ranging from hexagonal to octagonal, with at least one instance of a stellate configuration. As in other Seljuk funerary monuments, brick is the building material of choice at Kharraqan. The bricks are set in mortar and laid to form an elaborate array of symmetrical patterns that play with repeated themes of projection and recess, light and shadow, solid and void, offering numerous variations on a theme that plays continually with the passage of light without the addition of color in the earliest examples. The tombs at Kharraqan conform to the characteristics of these early examples, which pre-date the emergence of the use of color in glazed bricks, faience mosaic, and ceramic tile. The extraordinary number of patterns on the later tower (for dating, see below) at Kharraqan is unrivalled within the corpus of elaborately decorated Seljuk brick monuments [32; 16].

3.3 Historical Inscriptions

Each building bears an Arabic inscription that gives its date of construction. The earlier tower, to the east, was built in 460H/1067-68AD. To the west of this, the later tower was built twenty-six years later, in 486H/1093AD. The historical inscriptions each contain, in addition to a date, the name of the architect (who seems to be the same person for both monuments), and an additional name, which is not entirely legible. In each case, this is presumably the name of the deceased, which has been tentatively read in a manner to suggest individuals of Turkic ethnicity [31; 8].

3.4 Qur'anic Inscriptions (see appendix)

In addition to an historical text, a Qur'anic inscription encircles each building. Although discrepant in date by more than a quarter of a century, the buildings are informed by the identical passage (59:21-24, app. A), executed in cut brick laid and measured carefully so as to circumscribe the entire structure, uniting all eight facets. This Qur'anic passage is surmounted by a frieze consisting of a rectangular panel on each face, which is also patterned in brick.

The later tower, to the west, bears a similar disposition of text, circumscribing the eight faces and buttresses and bears a second Qur'anic text (23:115, app. B), also in cut brick, which frames the entryway below the historical inscription. Such Qur'anic inscriptions are generally accepted as formulaic; the meaningfulness in the choice of their selection is often explained in rather general terms [31; 8]. In the case of these monuments, one author [31] notes the lack of the occurrence of these verses on other funerary monuments, but cites the appropriateness of the preceding verse (59:20), which is here omitted. Another author relates the general content of these verses (God's attributes, and sending down the Qur'an) to inscriptions on mausoleums elsewhere [8]. But a closer consideration of the selected texts suggests that this

passage was purposefully chosen for the eastern, earlier monument, and repeated purposefully on the later, western monument. The specificity of the vocabulary suggests that the texts actually refer to the patterns, and may serve as verbal analogues to the associated visual images of abstract patterns, helping to inform our discussion of these monuments in an effort to ascertain their meaning for contemporary viewers at the time of their construction. If so interpreted, the patterns become visual renderings of the Qur'anic text and relate to what is provided to mankind for reflection.

4. Meanings and Interpretations of Geometric Patterns

4.1 Traditional Interpretations: Decorative, Symbolic, Aesthetic, Analytical, Spiritual

In the consideration of Seljuq monuments on the Iranian plateau, and Islamic art more broadly, several interpretations of geometric patterns have been offered. A rather typical interpretation assumes a decorative intent to embellish, occasionally with a formal acknowledgment of the role of geometry in organizing space [3]. It has also been suggested that geometric pattern and interlace may make allusions to paradise, while serving an apotropaic protective function [13]. Visual metaphors of paradise are also posited for the symbolic images of peacocks and pomegranates, painted on the interior of the earlier tomb tower, considered especially appropriate to the funerary context [12]. The Qur'anic inscriptions at Kharraqan, as elsewhere, are generally treated as formulaic, related broadly to themes concerning death and burial [8]. Several authors recognize a generic visual relationship of geometric patterns in brick to those of textiles [14; 15; 27] and have proposed the notion of an operative textile aesthetic for early Islamic contexts. Other authors offer a more analytical interpretation of the geometry itself [11; 9; 1; 28], some relating geometry to issues of spiritual concern [11], linking the multiplicity of forms to the doctrine of *tawhid*, or Unity, in which God is at once one and unified, while present in many manifestations [4]. An aesthetic interpretation emphasizes the role of geometry and pattern in the construction of beauty, without cultural specificity [15:154].

4.2 Allegorical Interpretation

No one heretofore has proposed an allegorical interpretation of the meaning of geometric pattern, to provide a direct link with these patterns to the concept of *mithal* as philosophical allegory. *Mithal* in Arabic has to do with likeness or resemblance: it may be an example, a similitude, a parable, or an allegory. The word has been used in a manner that corresponds to allegory and has been so treated in Western scholarly literature in efforts to explicate mystical aspects of Islamic philosophy [10; 25].

Several observations are relevant before considering such an interpretation. The multiple patterns at Kharraqan are each used in a consistent manner to organize and fill a defined space. Each pattern exists in relation to others; that is, each rectangular unit, or panel, or arcuate unit, or arch, is one of eight such units, all equal. In and of themselves, the spatial divisions are not unique, and each pattern could conceivably replace any other within the defined spatial unit. Further, each of the patterns is conceived as an infinitely repeating pattern, delimited only by the limits of the space in which it is set. Since each of the patterns is infinite in its conception, rather than self-contained, each could also replace another in any spatial unit. Together, thus, they seem to represent something larger than the forms of their individual expression.

Taken together, the patterns are interchangeable. Each is not, then, a representation with a specific meaning, but rather pointing to something else. In this sense, geometric patterns might be construed much in the manner of the ninety-nine names of God. The "beautiful names" each point to God -- no single one of them is sufficient alone, but each of them is sufficient to point to Him. Perhaps it is not so coincidental that the Qur'anic passage selected to encircle both monuments contains numerous names of God, and refers specifically to the most beautiful names (*al-asma' al-husna).*

The first verse of the Qur'anic passage selected to encircle both Kharraqan towers (59:21), itself contains the word, *amthal*, a plural of *mithal*, which is generally translated as parable or similitude and is often directly associated with the Verse of Light (24:35) in which God's light is likened to that of a lamp in a niche. It seems reasonable to propose that the precise combination of this Qur'anic passage with these panels of geometric ornament, together, may inform the meaning of these monuments. In this sense, the range of geometric forms would visually articulate what was revealed to Muhammad and is written in the Qur'an. The artistic rendering would thus reify the abstract concepts of the Qur'anic verses. Does this linkage not

represent, literally as well as metaphorically, a specificity of intent, striking a chord with what is recorded both visually and verbally on the exterior surfaces of these buildings? Taken together, one rendering reinforces the other, contributing to a cogent architectural program, a cycle of significant meanings and spiritual values. Much like a parable or homily for the teaching of morals, or a philosophical or literary allegory, each geometric pattern would thus signify more than the surface appearance of its individual artistic expression.

If each geometric pattern itself serves as a *mithal*, an exemplar, each offers a likeness to one another, but also a resemblance to something else. Expanding upon this interpretation, these *amthal*, in the plural, are precisely here in this context what is provided to men for their reflection. In this sense, the geometric patterns may be seen as a visual commentary on this Qur'anic verse, offering an allegorical interpretation.

If we turn to Islamic philosophy as it was being discussed at the time the Kharraqan towers were constructed, we find potentially strong support for this interpretation. Avicenna (Ibn Sina), who died in 1037 A.D. in Hamadan, had traversed this region and his philosophical writings were well known in Iran at this time. One of his main concerns in metaphysics was the difference between essence and existence. He called light "the visible in essence, and the cause that changes whatever is potentiality invisible into actuality" [20:113], and related this role to universal forms and the relation of material intellect to prime matter and potentiality. Let us consider the visual aspect of the tomb towers at Kharraqan at dawn, as the sun appears in the east and illuminates the brick patterns, which project slightly beyond the beds of mortar. As the sun rises, more and more of the patterns are illuminated and defined, being brought from potentiality to actuality. Avicenna further relates the importance of the niche, "the walls of a niche are close to each other and it is thus excellently predisposed to be illuminated since the closer the walls of a place are to each other, the greater the reflection and the light it holds" [20:116]. This commentary is in relation to the Verse of Light (Qur'an 24:35), in which God's light is likened to the light from a lamp in a niche (see above). This verse also states that "God strikes similitudes for men" and "God has knowledge of everything." The actualized intellect is then likened to light, light moving it from potentiality to actuality [20:116], and the universal intellect is said to "consist of the forms of numerous universals…not essentially but accidentally, acquiring its oneness from Him who is essentially one" [20:117-18].

This discussion continues, making reference to a statement of Muhammad to the effect that "each soul after death is resurrected"[20:118]. The journey of the soul goes through a series of stages, "The external senses …apprehend the sensible form together with its matter…there is the faculty that judges these forms in a logically possible way…Then there is another, the faculty that judges these forms in a necessary way: This is the intellect. These add up to 8. Now, when all the faculties are present, they lead to eternal happiness and entry into paradise" [20:120-21].

As an Arabic word, *mithal* also came to be used in Persian texts in classical Islamic times. The *alam al-mithal* (*alam-e mithal* in Persian) is an imaginary realm [10; 25]. Corbin characterizes it as a "topography of visionary experience" [10:8], an intermediary place, somewhere between the realm of the visible and the idealized, between the empirical world and that of the abstract intellect. Corbin asserts that the forms and figures of what he calls the imaginal world (*alam al-mithal*) "do not subsist as the empirical realities of the physical world…nor [do they] subsist in the purely intelligible world" [10:10]. He alleges that they have a corporeality and a spatiality all their own, one that he characterizes as an "immaterial" materiality in comparison to the dimensionality of the sensible world. This visionary realm, which is, in effect, interdimensional, corresponds to what Corbin has called *mundus imaginalis*, "a world that is ontologically as real as the world of the senses and that of the intellect" [10:9].[2]

The character and quality of geometric pattern as visually expressed in Iranian brickwork in the Seljuk period seems to fit the descriptions of this imaginal realm, both conceptually and ontologically. If we think of geometric patterns as images or maps of the visionary realm – neither memory, nor depicting an objective reality, but as ideas generated by the mind – they become intellectual images, which are visionary but not visual. Expressed in this manner in brickwork, such images of the mind can be seen and perceived by the senses.

Corbin relates this realm to that of the soul: "This fully objective and real world with equivalents for everything existing in the sensible world without being perceptible by the senses is designated as the eighth clime…a clime outside all climes…a place outside all places, outside of where (*na koja abad*)" [10:9].

4.3 Epiphanic Interpretation: For the Glorification of God

Once we accept the possibility that each of these patterns is a *mithal*, they are removed from the artistic realm of representation. This presents us with a not inconsiderable series of ideas with which to concern ourselves at another time. Let it suffice for the present to consider, however, several possible further meanings. If taken literally, the panels of geometric ornament themselves may be seen as similitudes by which men may reflect. Sometimes, this is referred to in philosophic sources "as if in a mirror." The notion of a mirror is indeed applicable to the properties inherent in symmetries based on reflection, which are physically present in the patterns as presented in cut brick. Are these *amthal*, then, themselves to be seen as manifestations of the divine, or are they part of God's creation, which has a separate ontological status? If the former, they may be considered as parallel to the most beautiful names, each calling upon us to reflect upon his nature. If the latter, they may each be considered as evidence for God's creation.

We may find further direction in our search by returning once again to the Qur'anic inscriptions on the monuments of Kharraqan. The final verse on both towers talks of the most beautiful names of God, and follows this with a rhetorical statement about all that is in heaven and in earth declares His praises and glory (59:24). Can we not, therefore, perceive all of these geometric patterns as being among God's creations, and thereby explain them as statements of praise, expressed visually for the glorification of God? Each panel of ornament, then, assumes an epiphanic role, providing an intuitive perception or insight into the reality or essence of God's creation. The prosaic quality of each interchangeable pattern then assumes special significance. Although not so very different from one another, that very difference itself is precisely what essentially characterizes its individuation as an expression of God's creation.

This brings us to a final consideration of the second Qur'anic inscription, which appears only on the later, western tower at Kharraqan. This inscription asks two rhetorical questions. The second question, "[Did you not think] that you would not be brought back to us?" may relate to the soul and its resurrection at the time of death, which is eminently suitable for a funerary monument. The first question, "Did you not think we created you in jest?" might similarly relate to the soul. But here, we might take this question literally, as with the quotation about *amthal*, discussed above. In this instance, let us consider the selected excerpt in relation to the tympanum above the entryway, thereby relating this inscription, too, to geometric pattern. The pattern depicted on the tympanum is highly unusual for, although it is periodic, it contains pentagons, which do not normally tessellate. The pattern bears three geometric shapes, which taken together form a tessellation (a square that is divided into four; a pair of triangles, each divided into three parts; and a pentagon that is divided into five parts). The possibilities in which a pentagon may tessellate are very few, and we might infer here a referential meaning in the question, "Did you not think we created you in Jest?"

5. Geometry in Islamic Art: Uses and Meanings

Relating geometric patterns to the plural, *amthal*, as used in the Qur'an, these buildings may serve to inform our understanding of a humanistic endeavor that brought to bear interrelationships among art, religion, and scientific knowledge in their time to articulate a sacred geometry. Such interpretation has significant implications for further understanding the predilection towards geometric patterns that are so pervasive in arts throughout the Islamic world.

Art, typically, is an area of human endeavor that, in particular instances, seeks to address the unknown and the unknowable. Art at times strives to depict that which resists representation – the mysteries that lie beyond human knowledge and experience, those spaces and places where science and religion intersect. Art as a humanistic exercise can accommodate ambiguity, whether in poetry, music, literature, or visual forms of expression, helping to forge understanding and to bring awareness where none previously existed.

The approach to understanding geometric patterns at Kharraqan is not so much about the *role* of geometry in Islamic art, but about *geometry* in Islamic art and its meaning for then contemporary viewers and pattern-makers. The questions are not about decoration and ornament, but about surfaces and the plane, about units and repeats, about the nature of the circle and space, finiteness and infinity, and the nature of God. What has heretofore not been adequately recognized is that the forms of these patterns, no matter how

basic they are now understood to be, were known empirically but not yet understood theoretically. In this sense, specifically, they were new not only to the artistic repertory of architectural ornament in the Islamic world. But also, the experiential play with pattern was new, as far as we yet know, in its relation to the exposition of Islamic philosophical discourse. Kharraqan may possibly be the earliest occurrence of just such a connection, one that may be further documented in later monuments of Seljuk funerary architecture on the Iranian plateau (such as Gunbad-i Surkh and Gunbad-i Kabud at Maragha [30:74-85]). At Kharraqan, the patterns seem to represent a particular exuberance, expressive of the thrill of setting forth an algorithm and seeing it through to completion within a given defined space. The effort exerted to produce the many patterns on the surfaces of these two buildings, and the greater effort still in the display of patterns in the later tower, suggests an intensity and intentionality, if not entirely methodical, that was destined to explore the mathematical properties of two-dimensional space. Beyond the physical properties of the spatial dimension, the evidence from Kharraqan points to the possibility that at a metaphysical level, an interdimensionality of visionary space may also have been the subject of exploration.

Questions regarding the nature of the divine, human beings in relation to the cosmos, the role of the intellect, and the ontological status of geometry, continue to inform our discourse today [5; 29]. Does geometry exist independent of human thought? Does geometry have an ontological reality within the cosmos, or is it a construct of human intellect? Raised initially centuries before the advent of Islam, these questions were not only being asked in the 11th century in the Islamic world, but answers were found, which seemed suitable to put forward on the tomb monuments at Kharraqan.

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Appendix A

Qur'an, ch. 59, v. 21-24 (tr. A. Yusuf Ali, *The Holy Qur-an: Text, Translation and Commentary*, The Islamic Center, Washington, DC, 1978, pp. 1527-28):

Had we sent down
This Qur'an on a mountain
Verily, thou wouldst have seen
It humble itself and cleave
Asunder for fear of God
Such are the similitudes *(amthal)*
Which We propound to men
That they may reflect.

God is He, than Whom
There is no other god;
Who knows (of all things)
Both secret and open;
He, Most Gracious,
Most Merciful.

God is He, than Whom
There is no other god;

The Sovereign, the Holy One,
The Source of Peace (and Perfection),
The Guardian of Faith,
The Preserver of Safety,
The Exalted in Might,
The Irresistible, the Supreme:
Glory to God!
(High is He)
Above the partners
They attribute to Him.

He is God, the Creator,
The Evolver,
The Bestower of Forms.
To Him belong
The Most Beautiful Names:
Whatever is in
The Heavens and on earth,
Doth declare
His Praises and Glory:
And He is the Exalted
In Might, the Wise.

Appendix B

Qur'an, ch. 23, v. 115 (tr. A. Yusuf Ali, *The Holy Qur-an: Text, Translation and Commentary*, The Islamic Center, Washington, DC, 1978, pp. 893-94):

Did ye then think
That We had created you
In jest, and that ye
Would not be brought back
To Us (for account)?

Notes

[1] This distinction was missed by Blair [8], who refers to the right-hand boss as an octogram, incorrectly. After 1966, the right-hand boss must have been removed; its absence is noticeable in photographic documentation by Seherr-Thoss taken in 1968 (Freer Gallery of Art Photographic Archives).

[2] Approximately synonymous with *alam-e mithal* in Persian is *alam-e khayyal*, a concept drawn uopn by S.H. Nasr with reference to later Persian painting to explicate pictorial space in relation to literary expression [21:179-82].