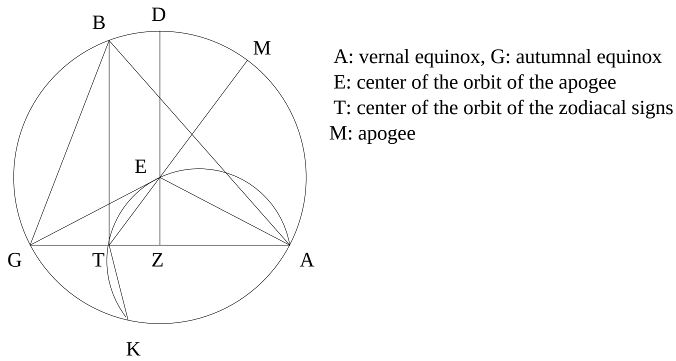
A computation from Chapter 7 of Book 6 of al-Bīrūnī, al-Qānūn al-Mas^{c}ūdī.[1]

Words in parentheses are my explanatory additions.

"I found in Jurjaniyya in Khwārizm in the year three hundred eighty five of the Yazdgerd (era)[2] the duration of the spring as $93;28$ (days)[3] and the duration of the summer as $93;8$ (days).



We repeat from the previous figure what we need.[4] Then, according to what we have found,[5] arc AB is $92;7,11,2$ and arc BG is $91;47,31,30$.
We circumscribe around triangle AET a circle. We join TK, AB, BG, EG. Then the sides of triangle ABG are known because the chord[6] AB is $86;24,27,39$ and the chord BG is $1,26;10,9,4$ and the chord AG is $1,59;55,47,44$.
If we divide the difference between the squares of AB and BG by the base AG, half the sum of this quotient and AG is $1,0;8,11,28$, and this is AT. And half the difference between the quotient and AG is $59;47,36,16$ and this is TG, which is equal to TK.[7]

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[1] Compare the Hyderabad edition vol. 2, pp. 655-656. and the Russian translation in vol. 2, p. 46. I have redone the computations and have corrected a few scribal errors in the first and second sexagesimals, but it is likely that there are scribal errors in the last sexagesimals which I have not found.

[2] The year 358 of the Yazdgerd era lasted from 30 Ramaḍān 406 /March 12, 1016 until 10 Shawwāl 407/March 11, 1017.

[3] al-Bīrūnī computes in the sexagesimal system, so $93;28$ means $93 + \frac{28}{60}$.

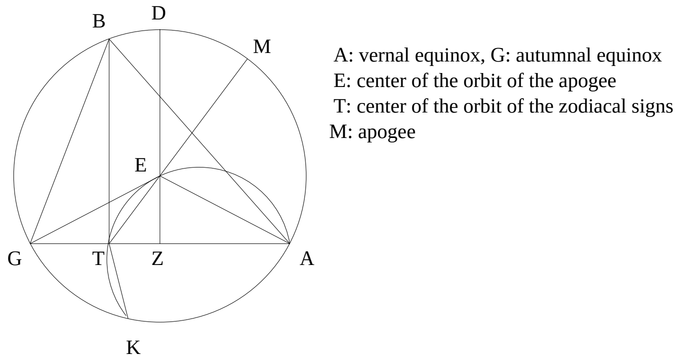
[4] ABG is the orbit of the sun around the center E. Point T is the center of the earth. The vernal equinox is A, the summer solstice B, the autumnal equinox G. Then ATG is the intersection of the plane of the solar orbit and the plane of the equator, so it is a straight line. Line BT is perpendicular to ATG, and the perpendicular DEZ is drawn to AG.

[5] Apparently al-Bīrūnī worked with a mean solar motion of approximately $0;59,8,8$ degrees per day. This factor times the number of days in the spring and summer is the length of the arcs AB and BG in degrees.

[6] The radius of the circle is taken as 60. The chord of an arc α is 120 times the modern sine of the angle $\alpha/2$.

[7] We have $TG{=}TK$ because circle $AETK$ passes through the midpoint E of the other circle.; note $\angle EGT = \angle EAT = \angle EKT$. From $AT = 1,0;8,11,28$ and $TG = 0,59;47,36,16$ it follows that $TZ = 0;10,17,36$. The same number can be found from the formulas $TZ = 60 \sin \text{arc} BD$, arc $BD = \frac{1}{2}$ (arc AB - arc BG).

But ATK is a broken line (inscribed) in arc $AETK$, which is bisected at E. So the square of AE is equal to the square of ET plus the product AT times TK. If we multiply AT by TG and subtract the product from the square of AE, the total sine,[8] the remainder is the square of ET.
Again, the square of ET is less than the squares of AE, AT by twice the product TA by AZ. So if we subtract from the sum of the squares of EA, AT twice the product TA by half AG, the remainder is the square of ET.
Again, the square of EG exceeds the squares of ET, TG by twice the product GT times TZ. So if we subtract twice the product of TG by TZ, the difference between the two results,[9] together with the square of TG, from the square of the total sine (EG), the remainder is the square of ET.
By each of these three procedures, the length[10] of ET is found[11] $2;3,26,24$, and the maximum equation is $1;57,54,1$.[12] The ratio of ET to TZ is as the ratio of the sine of the right angle Z to the sine of angle ZET. So angle ZET is $4;46,59,21$. We extend TE in a straight line towards M, that is the position of the apogee. So it becomes arc MD, so this arc is known, and $<$ the sine of $>$[13] arc DB is $0;10,17,36$, so the whole arc MB is known. The amount of equation which belongs to is is close to $0;9,55$.[14] Thus the position of the apogee is in Gemini[15] $25;3,5,24$.



[8] The sine function is defined in a circle with radius AE=60.

[9] Perhaps al-Bīrūnī means $2TZ = AT - TG$.

[10] One can also find $|ET|$ by the theorem of Pythagoras from $|TZ|$ and $|EZ|$, which can be found from $|EZ| = \sin\frac{1}{2}(\text{arc}\,AB + \text{arc}\,BG - 180^o) = 2;2,52$. Al-Bīrūnī's method can also be applied to related but more difficult astronomical problems.

[11] The Arabic edition and Russian translation have $5;2,3,26,24$.

[12] The "maximum equation" is the maximum angle $\angle EXT$ for points X on the circle. The maximum is reached when TX is perpendicular to TE.

[13] Because al-Bīrūnī means $|TZ|$ here, I have restored $<$ the sine of $>$ to the text. The word is omitted in the Hyderabad edition and in the Russian translation.

[14] This is half of the difference between arcs AB and BG.

[15] The Arabic edition and Russian translation have $25;13,5,24$.