The mathematical methodology of Islamic astronomy

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Medieval Islamic Astronomy (130/750 - 1165/1750)

Leading questions of this talk:
a. what mathematical knowledge did the medieval Islamic astronomers need?
b. what obsevations did they use?

1. Solar motion.

2. Lunar motion.

3. Sizes and distances of celestial bodies.

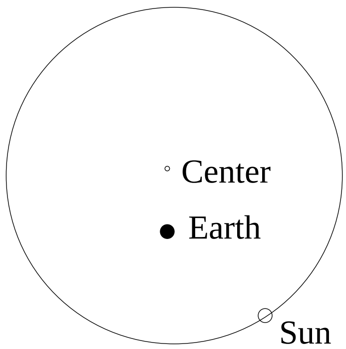
4. An example: the solar theory of al-Bīrūnī.

Observations and theory in Islamic solar theory

Assumption: the sun moves uniformly in a circle around a center which does not coincide with the Earth.

Input: Daily observations of the solar noon altitude. (This is high in summer, low in winter).

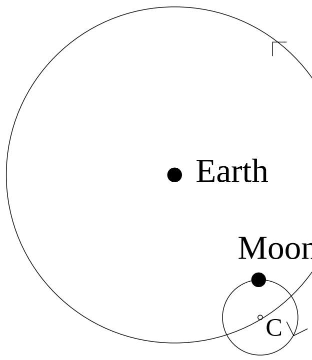
Output: a theory of the solar motion.



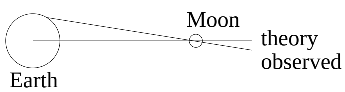
This had been done by Hipparchus and Ptolemy, and was refined by Islamic astronomers.

The lunar theory.

Assumption: the moon moves uniformly on an epicycle around a point, which moves uniformly in a circle around the <u>center of the</u> earth.



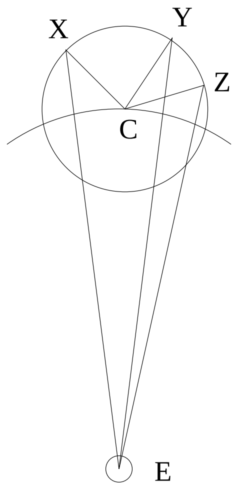
Problem: The moon is so close that we cannot ignore its parallax: the difference between the position seen from the center, and the position seen by the observer.
So we almost never know where the moon is with respect to the center (except when it is right above our head).



Solution: look at triples of lunar eclipses (close in time).
Observe the midpoint carefully.
In the middle of a lunar eclipse, the position can be **computed** as sun + 180^o.

Three lunar eclipses

Input: the period of the velocity of the moon (varies between 11 and 15 degrees per day); the moments and positions of three lunar eclipses

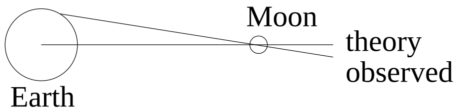


C is the center of the epicycle, X the position of the first eclipse. For the second and third eclipse, turn the epicycle back around E so its center coincides with C, and let Y and Z be the positions of the eclipses.
Time intervals give $\angle XCY, YCZ$.
Position differences give $\angle XEY, YEZ$.
Hard mathematics then produces $|EC| : |CX|, \angle CEX$.

One needs: very good geometry to understand this solution and do the computation.

Lunar Theory

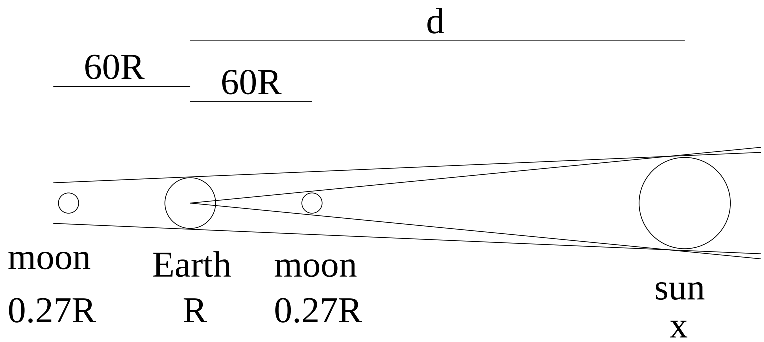
One can now compare the theoretical position with the actual observed position.
The difference is the lunar parallax q.



($\sin q = r/d \cdot \cos a$, a the altitude above the horizon, r the earth radius, d the lunar distance).
Hence $d \approx 60r$ (mean distance).
Lunar radius ≈ 0.27 earth radius.

Solar Distance

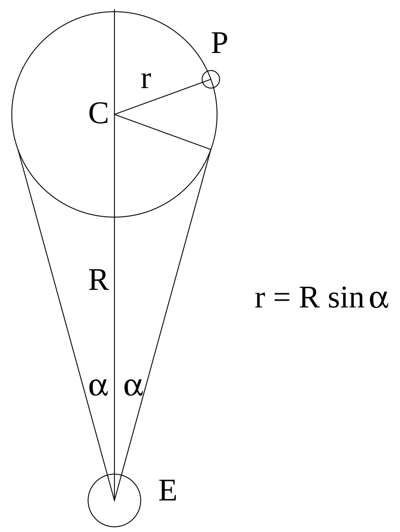
Input: radius of shadow cone of the Earth is 13/5 times the moon radius.
Apparent radius of sun and moon is $\frac{1}{4}^{o}$.



Output: solar distance is $\approx$ 1200 times earth radius,

“Proof” of the Ptolemaic System: Part 1

Basic idea: ratio between maximum and minimum distance of planet can be deduced from observations.



$$\max/\min = (R + r)/(R - r) = (1 + \sin\alpha)/(1 - \sin\alpha).$$

“Proof” of the Ptolemaic System: Part 2

Maximum distance of the moon us 64 earth radii (R).

Assumption: this is minimum distance of Mercury.

For Mercury, min:max = 34:88. So max. Mercury is 166R.

Assumption: this is minimum distance of Venus.

For Venus, min:max = 16:104. So max. distance Venus is 1079R.

But: the solar distance is approximately 1200 R; minimum 1160R.

So everything fits together!

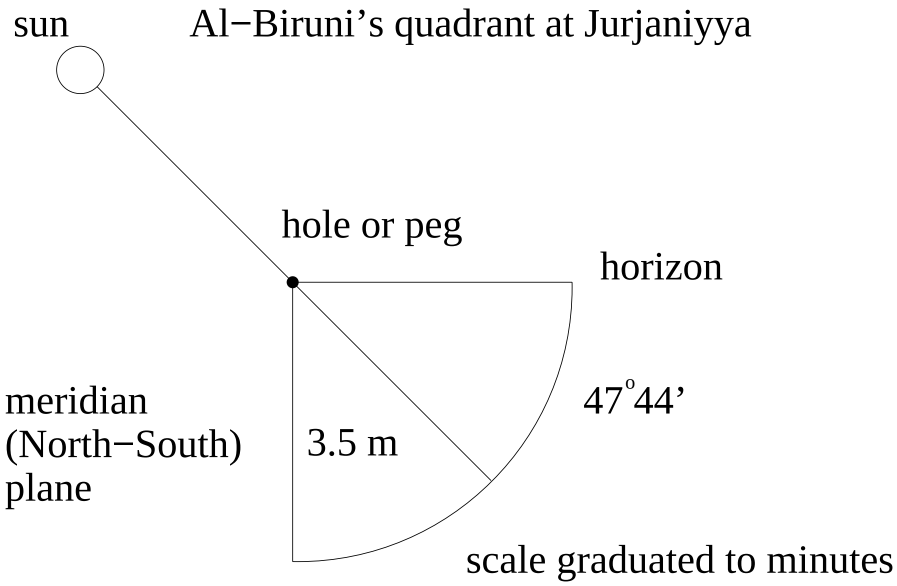
Rest of the talk: One specific example

The determination of the solar eccentricity and apogee by al-Bīrūnī, by means of observations he made in Jurjāniyya (now Konya-Urgench, Turkmenistan)

Source: al-Qānūn al-Masᶜūdī (mathematics)

details on observations in:
كتاب تحديد نهايات الأماكن في تصحيح مسافات المساكن
The Book of the Determination of the Coordinates of Positions for the Correction of Distances between Cities

Observations of solar noon altitude.



Results:

Maximum observed solar noon altitude at Jurjāniyya: $71^o18'$

(Not stated in sources: Minimum observed solar noon altitude at Jurjāniyya: $24^o8'$)

Average $47^o43'$ is 90^o minus the geographical latitude of Jurjāniyya, so the latitude of Jurjāniyya is $42^o17'$.

Obliquity of the ecliptic is $(71^o18'$ - $24^o8')/2 = 23^o35'$.

(Al-Bīrūnī checked these results by other methods as well.)

Observation of autumnal equinox

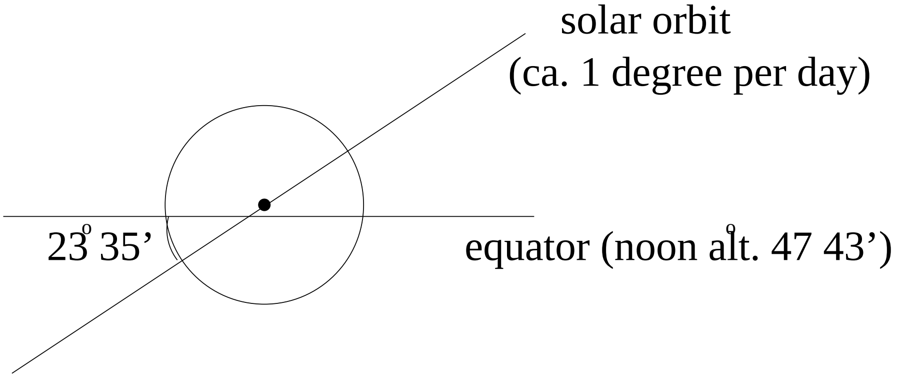
(Autumn begins at noon when the sun is at noon altitude 90^o minus latitude, is $47^o 43'$).

Al-Bīrūnī says:
In Jurjāniyya,at the Emirate House,
I observed the noon altitude of the sun on

Monday 11 Rabī^c II, 407 H.
= Mihr 10, 385 Yazdgerd =
Aylul 17, 1327 Alexander

I found $47^o 44'$.

He concluded that the autumnal equinox (noon altitude $47^o43'$) was 1 hour after noon. Reason: one day later, the noon altitude would be $47^o20'$.



sin(23°35’)=0.4001

change in altitude: 24’ per day.

change in longitude: 60’ per day.

Similarly: vernal equinox (beginning of spring).
Result:

Solar solstice requires interpolation between noon altitude observations made during many successive days.

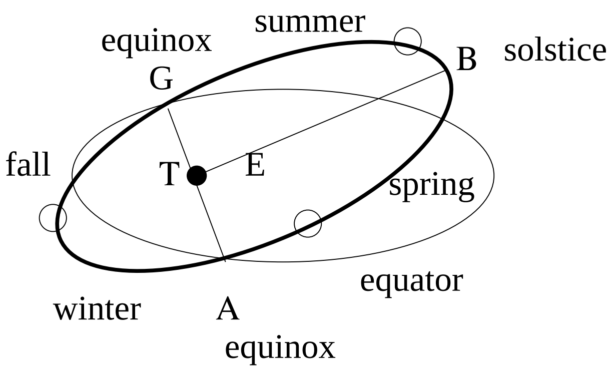
Al-Bīrūnī found the length of spring as $93 + \frac{28}{60}$ days

the length of summer as $93 + \frac{8}{60}$ days

Sexagesimal Notation:
I will use 93;8 and 93;28
Al-Bīrūnī used صج ح and صج كح

Solar orbit

Circle, around a center E, which does not coincide with the earth.

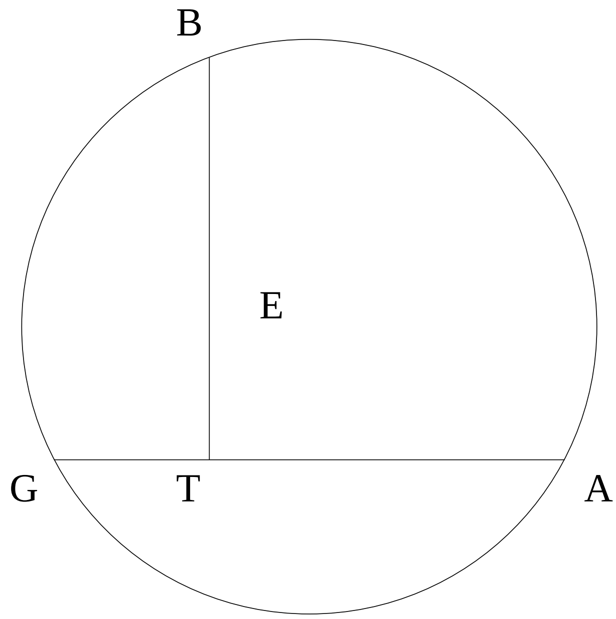


BT perpendicular to ATG

Question:
Where is E and what is its distance to the earth?

The answer is needed for the prediction of the position of the sun at any moment.

Al-Bīrūnī's computation 1.

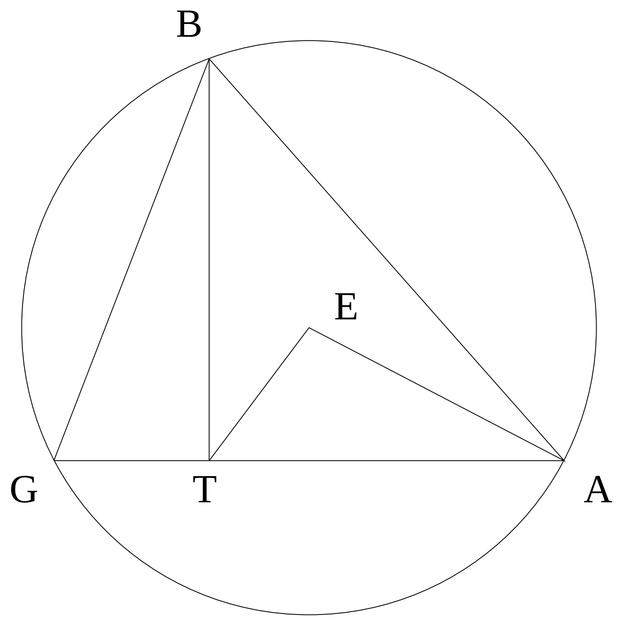


$AB = 93;28$ days, $BG = 93;8$ days.
Mean solar motion is $0;59,8,8$*? per day.*
So $AB = 2;7,11,2$? degrees and $BG = 91;47,31,30$? degrees.

Take radius = 60. Then $|AB|$ = 86;24,27,39?, $|BG| = 86;10,9,4$?, $|AG|$=119;55,47,44? See handout.
Sine table is necessary!

BT is altitude in triangle ABG. Therefore ...
$|TG| = 59;47,36,16$, $|AT| = 60;8,11,28$.
(Hence question mark in $|AG|$ can be removed).

Al-Bīrūnī's computation 2.

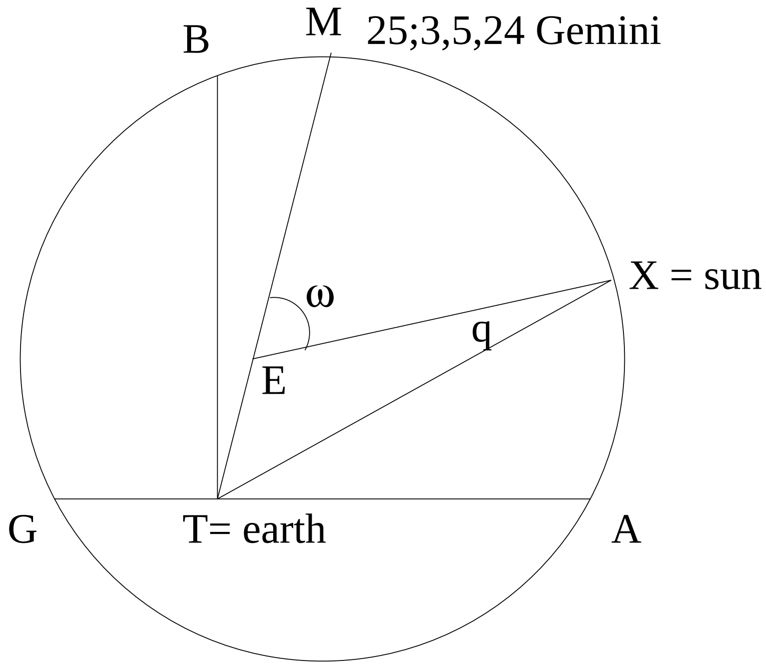


E is center of circumscribed circle of triangle ABG.
$|ET|^2 = |AE|^2 - |AT| \cdot |TG|$, and $|AE| = 60$.
Result in Arabic edition and Russian translation: $|ET| = 5; 2, 3, 26, 24$.
Edition is wrong. Exact result is $|ET| = 2; 3, 26 \ldots$.

Conclusion: al-Bīrūnī said $|ET| = 2; 3, 26, 24$.

More mathematics (see handout):
$\angle ETB = 4; 56, 54?$
Point B is end of sign Gemini. So E in direction $25; 3, 5, 24?$ Gemini.

On the basis of this information, we can compute the position of the sun at any moment:



ω is linear function of time.

Then compute angle q from ω, $EX = 60$, $ET = 2;3,26,24$.

Position from the earth:
$\angle MTX = \omega - q$.

There were many applications of solar theory:

determination of the length of day,
determination of the time of day from the solar altitude at a certain instant
(spherical trigonometry needed)

(We will see a different way of solving this problem in the astrolabe workshop)

Conclusion: Medieval Islamic astronomers required lots of arithmetic, geometry, and trigonometry.