The mathematical and historical background of a newly discovered instrument for finding the direction and distance of Makka.

Jan P. Hogendijk and
Eelco Nederkoorn

Department of Mathematics, University of Utrecht,
Netherlands

Summary of the first half of the talk:

1. The discovery of the instrument, and its publication in 1999 by Prof. David King (Frankfurt)
2. How does the instrument work?
3. Why does the instrument work?
4. Origin of the instrument: European or Islamic?
5. Relationships with medieval Islamic mathematics
6. Possible references in Arabic manuscripts
7. Conclusion: This is an Islamic instrument.

In the second half of the talk, Mr. Eelco Nederkoorn will present his computer programmed version of the instrument.

One instrument discovered in 1989, another in 1995.

Provenance: Isfahan (Iran), 17th-18th c.

Published in D.A. King, *World-Maps for Finding the Direction and Distance to Makka*, Leiden: Brill, 1999, 638 pp.

A third instrument is now on display at Harvard University.

How does the instrument work?

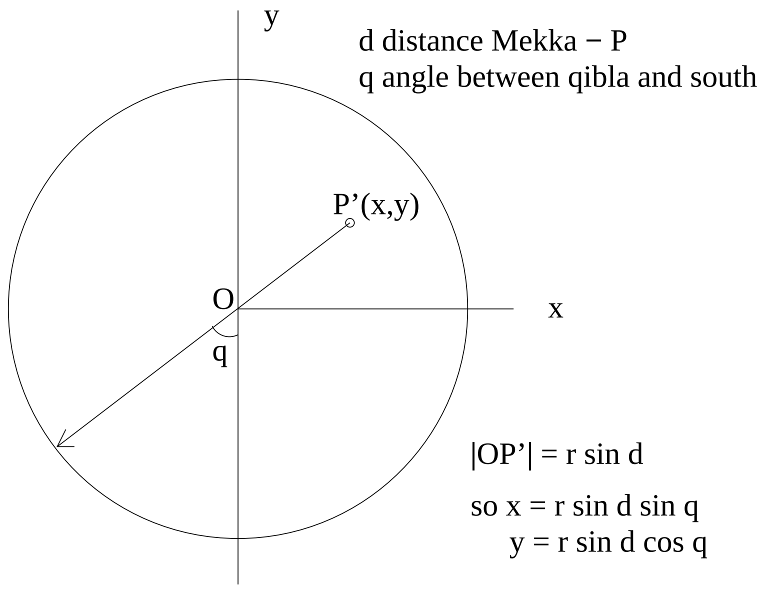
The grid is essentially a world map.

Put the ruler on your location on the map (be sure to choose the correct side, which passes through the center)

On the scale you see the direction of Makka in your locality

On the ruler you can read off the distance to Makka.

Some mathematics:



Why does the instrument work?

1. Consider the mapping f, which projects the earth on a (theoretically exact) instrument via:

$$f : (\Delta\lambda, \phi) \to (q, d) \to (x, y)$$

where

$\Delta\lambda$ longitude difference, ϕ geographical latitude of P, d distance Makka-P in degrees, q angle between qibla and South point, (x, y) coordinates of the "projection" of P on the dial of the instrument.

Investigate the images under f of:

the meridians ($\Delta\lambda$ = constant)

and the parallels (ϕ = constant).

Big question: Do the images that we get correspond to the grids on the actual instruments?

Answer $\approx$ Yes!

2. Details:

For some constant r which depends on the size of the instrument:

$$x = r \sin d \sin q, \; y = r \sin d \cos q,$$

If μ is the geographical latitude of Makka, by the sine and cotangent rules in spherical triangle NMP:

$$\sin q / \cos \mu = \sin \Delta\lambda / \sin d;$$

$$\sin \Delta\lambda \cot q + \cos \phi \tan \mu = \sin \phi \cos \Delta\lambda.$$

Hence, by the sine rule:

$$x = r \sin \Delta\lambda \sin \mu,$$

(the image of any meridian $\Delta\lambda = c$ on a mathematically exact instrument is a straight line parallel to the $y-$axis)

By the cotangent rule:

$$x^2 / \cos^2 \mu + (y + \tan \mu \cos \phi)^2 / \sin^2 \phi = r^2.$$

(Image of parallel for latitude $\phi = c$ on a mathematically exact instrument is an ellipse with centre on the $y - axis$, constant horizontal major axis of length $2r \cos \mu$, etc.; unless $\phi = 0$ or $\phi = \pm 90^o$.

Hypotheses on the origin of the instrument:

Sotheby's catalogue: "The projection is of western European inspiration ... and this unusual instrument is interesting as evidence of the assimilation of European science and technology in Persia in the 18th century."

King 1999: Ḥabash al-Ḥāsib (Baghdād, 9th c.) (on the basis of his genius and his discovery of the Melon-Shape astrolabe).

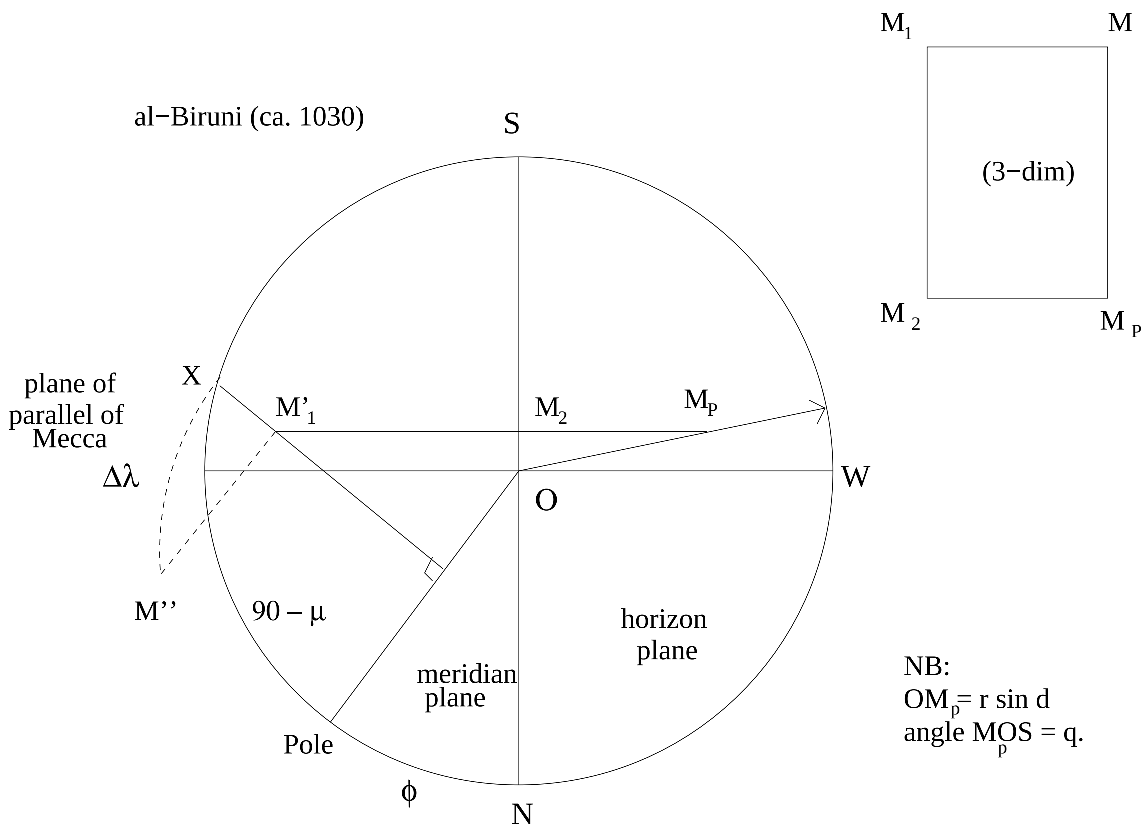
Elly Dekker (Utrecht), 2000:

"The newly gained [mathematical] insight into the projection itself, however, shows that a direct relation between the Iranian maps [i,e, the instruments] and Islamic mappings known from the ninth century, as suggested by King, does not exist."

"it seems premature to claim a medieval origin of the maps"

"Might [the discovery] have happened as part of the as yet little studied cultural exchanges between Europe and Persia which took place in the seventeenth and later centuries"?

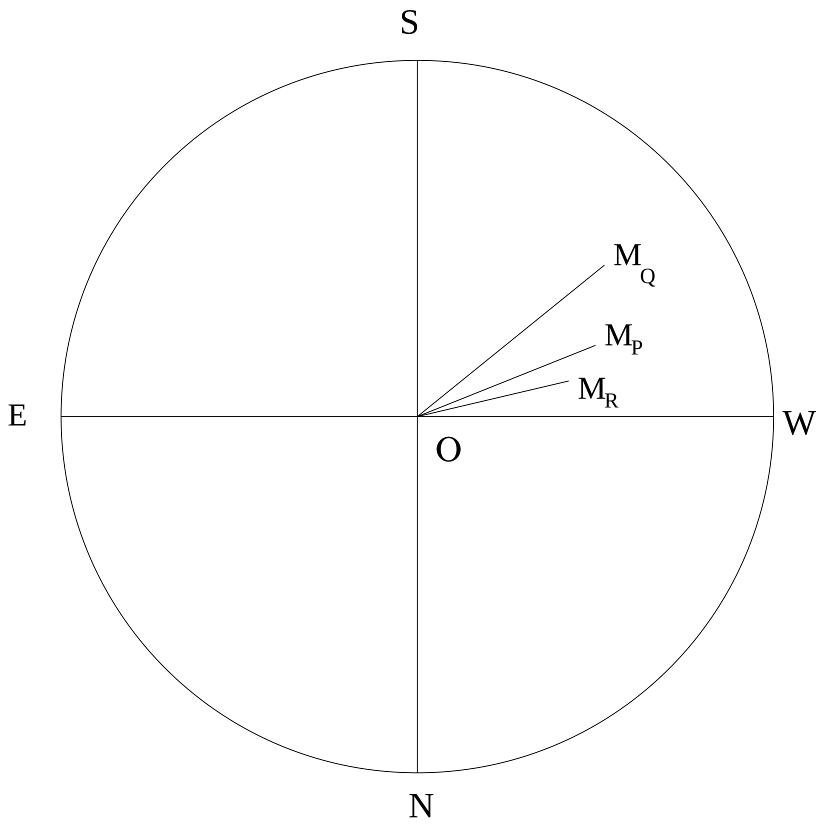
Islamic analemma construction of the direction of Makka (3d/9th century, and later)



Repeat this construction for different cities:

World−map:

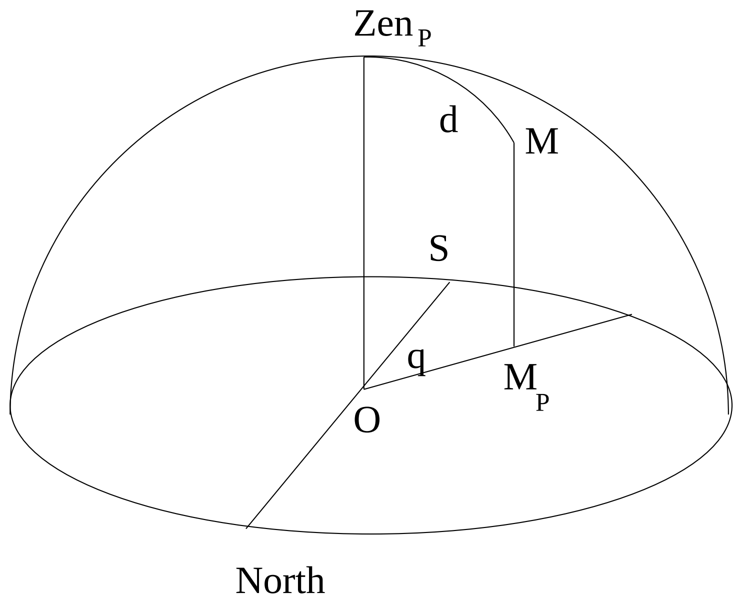
Repeat the construction of the point M_P for other localities Q, R etc. in the same figure. Call M_P the projection of P, M_Q the projection of Q, etc.



Now interchange N with S, and E with W.

Now we get the instrument!
But how do we get the grid?
If ϕ=constant, how do we get ellipses?

M_P is the projection of the zenith of Makka on the horizon.

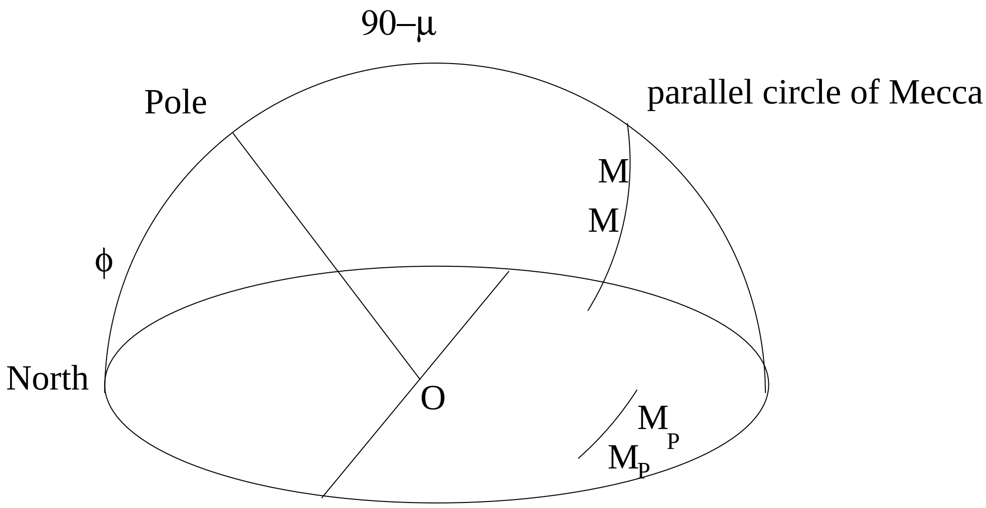


Note: OM_P= r sin d
angle SOM_P = q

Compare with the ruler on the instrument!

If ϕ = constant, the zenith of Makka is on a circle!

Why are the curves on the grid ellipses?



If ϕ is fixed, the celestial North pole is fixed.

Then M is on a fixed parallel circle (independent of $\Delta\lambda$)

Thus M_P is on the orthogonal projection of this circle (an ellipse).

Possible traces in Arabic manuscripts

Anonymous summary of Apollonius, *Conics* in the revision of Abū Ja^{c}far Muḥammad ibn al-Ḥusayn al-Khāzin

The author says: "This amount of knowledge on conic sections is sufficient for the knowledge of the paths of the tips of shadows of gnomons (in sundials) on every day of the year, **and it is useful in the derivation of the azimuth of the qibla by means of conic sections** (istikhrāj samt al-qibla bi-ṭarīq quṭū^c al-makhrūṭāt)"

The author also mentions the trisection (Thābit ibn Qurra and al-Harawī) and the chord of one-seventh of the circle.

2. Summary of a work by the astronomer Muḥammad ibn Aḥmad al-Khāzimī, who made observations in Isfahan in 453/1061.

This work had sections on:

* finding the azimuth of localities on earth by means of conic sections
* finding these azimuths in a practical way

The summary includes a proof that the orthogonal projection of a parallel circle on the celestial sphere on the horizon is an ellipse
(this proof shows that the curves on the grid of the instrument are ellipses)

Conclusion

This instrument has no relation to 17th-century European mathematics.

(The projection was discovered in the Western literature in 1968)

But the instrument can be obtained by repetition of a traditional Islamic geometric construction.

Hence it must be an Islamic instrument.

The grid can easily be found by al-Bīrūnī's analemma, not by earlier analemmas. Al-Bīrūnī does not mention the instrument. Perhaps it was discovered around 440/1050, after al-Bīrūnī's death, but before al-Khāzimī wrote his work.