Closed greenhouses

Final assignment for the

A-lympiad 2008

Garderen,
March 14 and 15, 2008

GUIDE FINAL MATHEMATICS A-LYMPIAD 2007

IN ADVANCE:

- ❑ First read the full text of the assignment so you will know what you have to do this weekend.
- ❑ One of the goals of assignment 1, designing a diagram, is that you yourself will understand the whole system of the closed greenhouse.
- ❑ Divide tasks where possible and consult when needed.

TIME MANAGEMENT:

- After breakfast on Saturday afternoon you will receive extra information, consisting of the characterisation of three days
- Keep an eye on the time you are using when you work on the different parts.
- Be sure to have enough time left to prepare your poster presentation on Saturday!
- You have to hand in your report on Saturday afternoon before 13:00.

HANDING IN:

The detailed answers for all parts. Take care that all information that is on your poster is also in the report!

The jury will receive copies of your work. Of course the copies must be legible. So use a black pen for writing and only print on A4 size paper. Any drawings that you hand in, must copy well, and it is therefore better not to use a pencil to make them.

JUDGING:

Among others, the following points are important for the jury:

- How complete the answers for the various parts are;
- The use of math;
- The argumentation used and how choices that have been made are justified;
- The depth to which the various assignments have been answered;
- The style of presentation: form, legibility, (copyable) illustrations etc;
- Originality and creativity.

Of all parts of the final, the two final assignments are counted as most important. The jury will also look at the link you make to the result from the first four assignments.

The closed greenhouse

Introduction

A lot of fruit and vegetables is grown in greenhouses. Especially in the Westland in the Netherlands there are many of these greenhouses, as you can see on this Google Earth picture.
The purpose of the greenhouse is to regulate the climate for the plants in it, so you are not dependant on the weather, or:

In a greenhouse you can set the temperature and amount of light you choose at any time.

The sun provides the light and lamps will provide light at night or in cloudy weather. The sun also provides heat: the temperature will rise significantly if there is direct sunlight on the greenhouse, which is where the term ‘greenhouse effect’ comes from. And if it’s not sunny, you can just heat the greenhouse. If it gets too hot inside, you can open the windows, and much of the heat will disappear again.
As a result, most greenhouses aren’t very energy efficient.

The latest development in greenhouse farming is the **closed greenhouse**.

In a closed greenhouse the windows are always closed. A great secondary advantage is that no pests can come into the greenhouse! The closed greenhouse also has a heating system that can also generate electricity. And the possibility to store heat in the groundwater.

The main question of this A-lympiad assignment is:

How, over the year, can you regulate the climate (temperature and light) in a closed greenhouse so that energy costs are as low as possible.

Regulating temperature and light

Temperature (heat) and light are regulated as follows in the closed greenhouse:
Light is generated by:

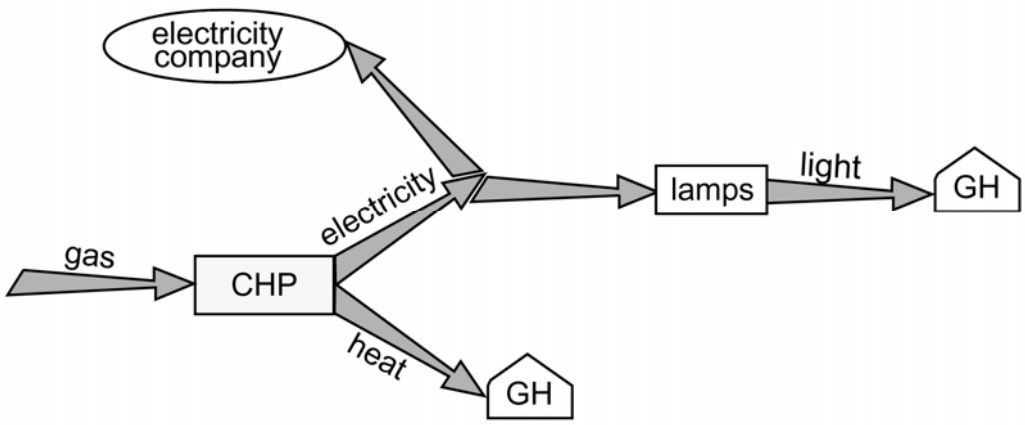
- The sun
- Lamps

Heat is generated by:

- The sun
- A CHP generator

What the sun does is obvious; lamps use electricity, which is provided by the electricity company. The CHP generator requires some explanation.

CHP stands for Combined Heat & Power. CHP generators provide both heat (and therefore function as a 'heater') and electricity. The CHP, as we will call it from here onwards, is gas-powered.
You can use the electricity generated by the CHP yourself, but you can also sell it to the electricity company if you don't need it yourself at that point. A diagram of the CHP looks like this:



All factors in the closed greenhouse system have now been introduced:
The sources: sun, gas company, electricity company
The machinery: CHP and lamps
and
the main actors: heat and light.

To get to know the system, you will first design two diagrams.

Assignment 1

a
Design a graphical diagram which includes all factors mentioned above and which clearly shows how light and heat get into the greenhouse.
b
Also design a diagram that shows how money, that is energy costs, flows through the system.

Use as little text as possible in both schemes, but so that someone without foreknowledge can still understand them.

Greenhouse in numbers

All factors that can play a part in regulating heat and light in the greenhouse are now known. To start your calculations, you have to know some values. The effects and the costs of heating cooling and lighting are given below.

Lamps

- There are 100 identical lamps in the greenhouse.
- One lamp uses 5 cents worth of electricity in an hour.
- All 100 lamps are on at night.
- 50 lamps are needed in cloudy weather and at dawn and dusk.

Sun

- The sun provides enough light for the greenhouse when the sky is clear.
- In clear weather the temperature in the greenhouse rises by 2 ^{o}C every hour.
- If, on cloudy days and at night, the temperature inside the greenhouse is higher than on the outside, the temperature will fall by 1^{o}C an hour (until the outside temperature is reached).
- In reverse: if the greenhouse is colder than the outside the temperature will rise by 1^{o}C an hour.

CHP

- When in use, the CHP, burns 10 euro in gas an hour.
- The CHP can raise the temperature in the greenhouse by a maximum of 3 ^{o}C an hour.
- The CHP, when it's on, provides exactly enough electricity to light all 100 lamps.
- The electricity generated by the CHP provides a maximum of 3 euro an hour if the electricity is sold to the electricity company.
- It is possible to have the CHP supply half its electricity for the lamps and the other half to the electricity company.

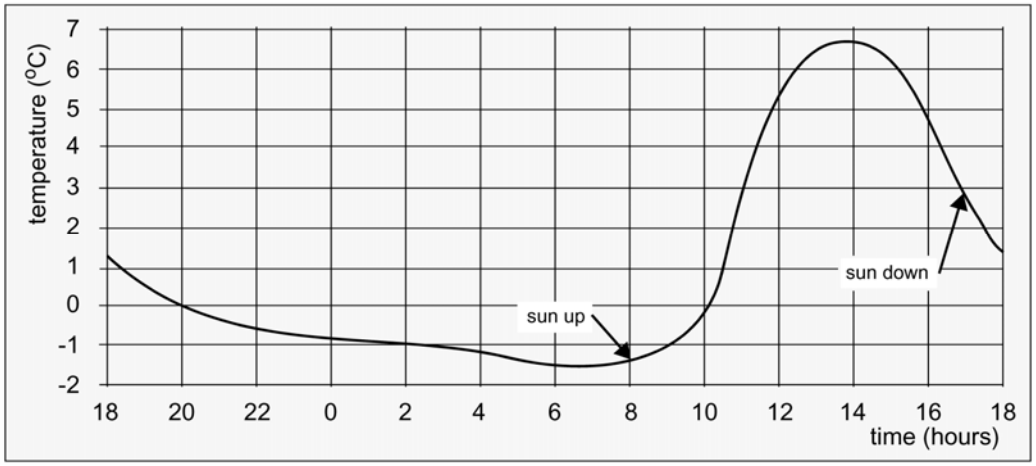
The first plants

The first plants grown in the greenhouse are tomatoes. Tomato plants grow best under the following circumstances:

- Daytime temperature of almost constantly 25^{o} C, night time temperature no lower than 18^{o}C
- Maximum amount of light both during the day and at night

The grower will try to realise these conditions by switching lamps and the CHP on and off. Of course he wants to keep his costs as low as possible. When and how long the CHP and the lamps are on will of course depend on a day's weather conditions and the continuously changing conditions in the greenhouse.
Here you see the weather conditions for a day in winter:

- Cloudy all day.
- The sun rises at 8.00 and sets at 17.00.
- The outside temperature follows the graph below.



What does the grower have to do with the CHP and the lamps to create suitable conditions in the greenhouse for his tomato plants for the whole day and night.

Assignment 2

In fact you will now do a simulation of the settings in the greenhouse; you will indicate what the CHP and the lamps do, and when they do it. Together those settings determine the temperature and the amount of light in the greenhouse. It is also your goal to achieve that as cheaply as possible.

a Night
At 22:00 the CHP is off, all lamps are on. At that time the temperature in the greenhouse is exactly 19 ^{o}C.
Design an on/off schedule for the CHP and the lamps, so that the temperature and the light remain at the correct values for the whole night (until 08.00 o'clock). Provide an orderly representation of this schedule, and also give the temperature in the greenhouse at each point in time.
Study a number of possibilities and try to find the cheapest on/off schedule.

b Day
Now extend your schedule with the daytime data (from 08.00 to 18.00).
Here too, study several alternatives to find the cheapest and therefore, as far as energy is concerned, most sustainable, way.

Keeping a cool head

Heating the closed greenhouse is done using the CHP. But it's not unimaginable that on an extremely sunny day the temperature becomes too high, higher than the outside temperature, even with the CHP switched off. This is the greenhouse effect. And you cannot open the windows.

The solution is provided by groundwater. We'll spare you the technical details, but what it comes down to is that you can use cold groundwater as cooling-water. To do that, you add a special type of pump to the system in the greenhouse.

Some technical data for this cooling system:

- The pump can lower the temperature by 2 ^{o}C an hour.
- The pump works on electricity, and uses € 4 an hour's worth of electricity.

Assignment 3

It is a sunny summer's day. The sun rises at 06.00 and sets at 21:30. The minimum temperature is 18,1 ^{o}C, and the maximum temperature is 30,6 ^{o}C.
Design an optimal (cheapest) on/off schedule for this sunny day (24 hours). Indicate for the lamps, the CHP and the pump when and how long they are on or off and what the temperature in the greenhouse is at any given time.
Show what your schedule costs and explain why it is the cheapest one.

Refining the model

According to physics, the rise and fall of the temperature when the heating is off and the sun doesn't shine doesn't quite work the way we describe the process here. The heat energy that is lost per unit of time does not in reality have a fixed value, but is in fact proportional to the difference between inside and outside temperature. In other words, if you don't heat the greenhouse, it will cool faster if there is a large difference in temperature with the outside.

In our model we translate this as follows:

The number of degrees that the greenhouse cools (or heats up) per hour without heating (or cooling) is one tenth of the difference in temperature.

Assignment 4

Look at the influence this refinement of the model has on the results of assignments 2 and 3.

Final assignment 1

a
Consider the (weather) conditions for four days over the year and see if you can find an on/off schedule to have the greenhouse operate as cost-effective as possible.
b
Of course there are some extreme circumstances which the machinery (heating and cooling) won't be able to handle. Describe as accurately as possible the (weather) circumstances for extreme winter and summer days where the greenhouse won't be able to provide the optimal conditions for the tomato plant.
c
Make a substantiated estimate of the energy costs over the year, if you grow tomatoes all year. Motivate all assumptions you make

Final assignment 2

By now you are an experienced 'greenhouse operator'!

On Saturday morning you will be given

- the weather characteristics for three different days
- the requirements the greenhouse will have to meet (regarding heat and light)
- the needed start values

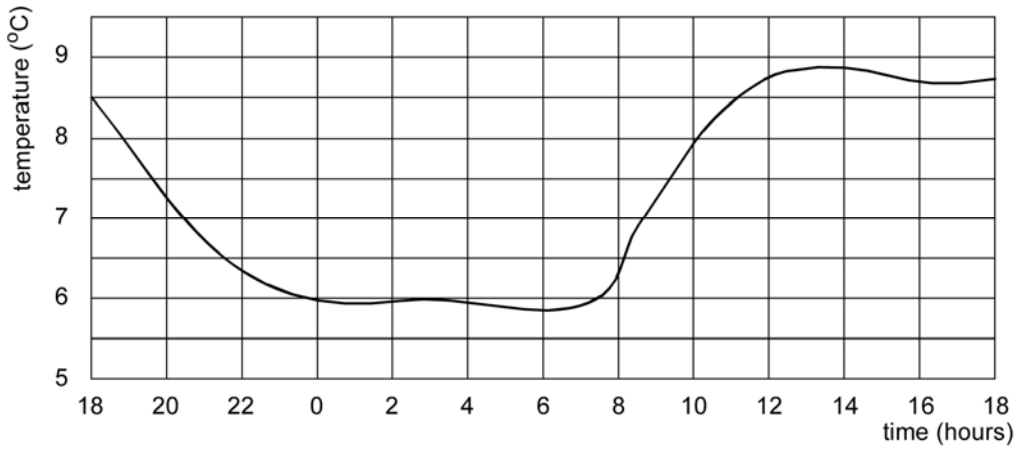
Determine the optimal on/off schedule for CHP, cooling (pump) and lamps for each day.

Presentation

The A-lympiad final finishes with a presentation of the results.
Make a poster which contains:

- The diagram from assignment 1, complemented with the cooling system
- An on/off schedule for one of the three days you were given on Saturday morning
- Total energy costs for the three days you were given on Saturday morning.

A day in October



Sunrise: 7.53 h
Sunset: 19:01 h

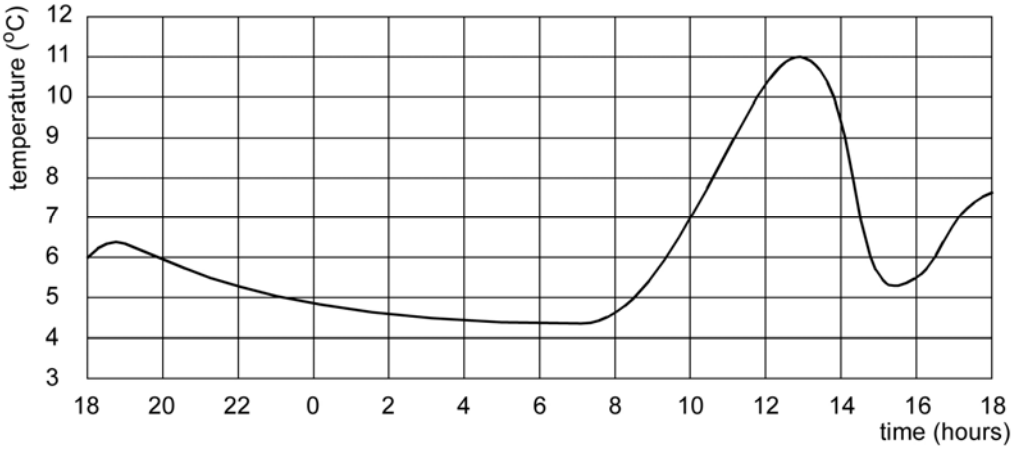
It was raining all day long. One of these boring autumn days that we know so well in the Netherlands.

At the moment the greenhouse is used for an experiment to grow a special kind of Japanese pear: the Nashi. The temperature is kept at 25 °C. Because it is an experiment, the grower tries to keep this temperature as steady as possible. The Nashi pear needs a lot of light.

The start temperature at 18.00 h in the greenhouse is 25 °C.

梨（NASHI）

A stormy day in March



Sunrise: 7.01 h
Sunset: 18:40 h

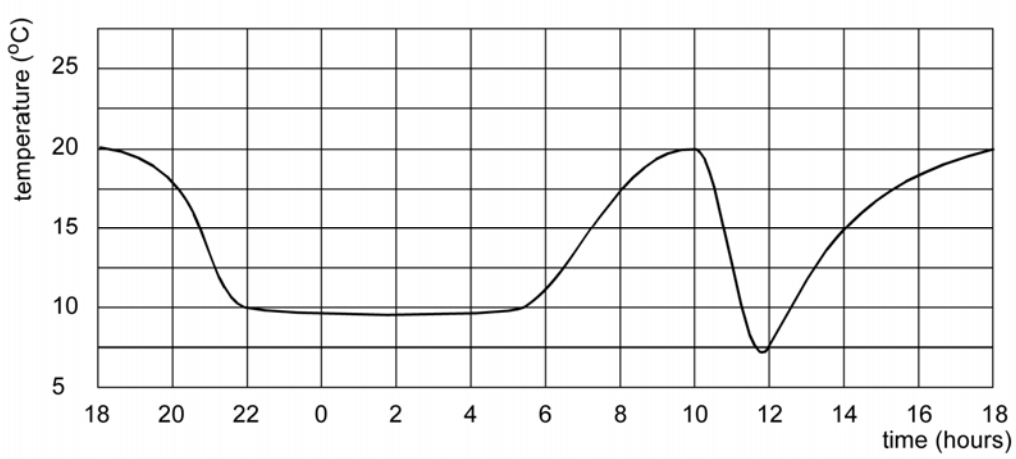
After a quiet night a sunny morning followed. At 13.00 h a severe depression with heavy weather and thunderstorms reached the country. After two hours the weather calmed down and even the sun returned for the rest of the day.

Another Japanese fruit, the Biwa, is grown in the greenhouse. In Japan it is an autumn fruit, so the temperature may vary from 15 °C to 25 °C. The Biwa also needs a lot of light.

The start temperature at 18.00 h in the greenhouse is 25 °C.

枇杷（BIWA）

An unstable day in May



Sunrise: 5.27 h
Sunset: 21.48 h

After a humid day a thunderstorm occurred at 20.00. This resulted in a relative cold night, followed by a sunny morning. But at 10.00 h another thunderstorm occurred . Fortunately, at noon the nice spring weather returned.

The Gooya, a cucumber like vegetable from Okinawa, the southern part of Japan, is growing in the greenhouse. The Gooya doesn't really have a preferred constant temperature; biologists think it grows best when there is a certain fluctuation in night and day temperature. In the greenhouse this is simulated with a difference between night and day temperature of 10 °C.

The start temperature at 18.00 h in the greenhouse is 25 °C.

ゴーヤ（GOOYA）

(Thanks to Minoru Othani, Kanazawa University, for the introduction to Japanese fruit and vegetables.)