Students' understanding of decimal numbers – a longitudinal study of learning

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Our decimals work - a summary

- Understanding how students think about decimals
- Tracing students’ progress in the longitudinal study
- Teaching interventions
- CD and website for teachers

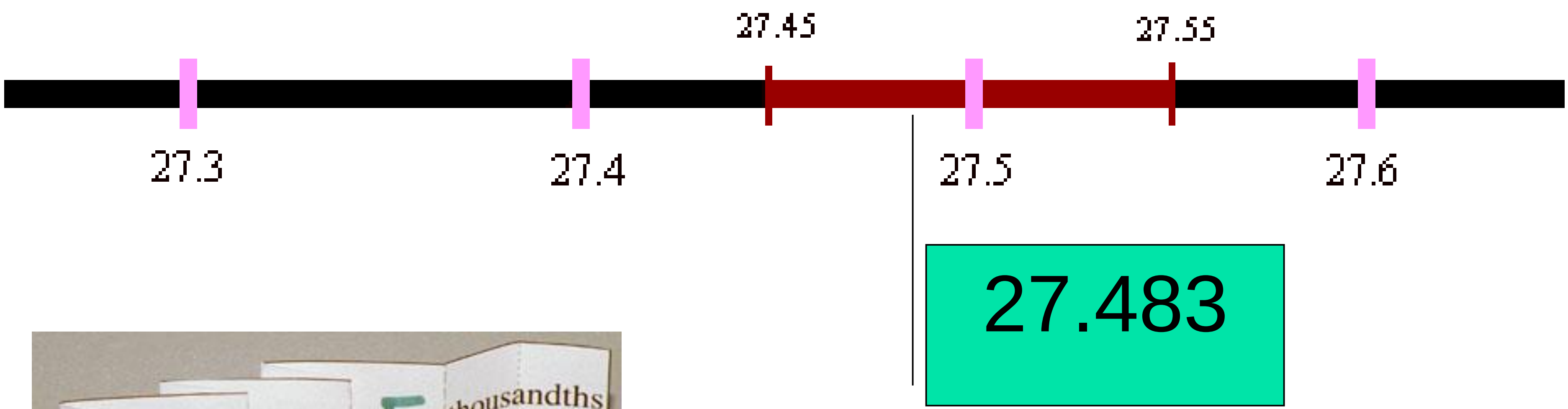
- Thanks to
  - Vicki Steinle
  - Liz Sonenberg
  - Ann Nicholson
  - Tali Boneh
  - Sue Helme
  - Nick Scott
  - Australian Research Council
  - many U of M honours students
  - Dianne Chambers
  - teachers and children providing data

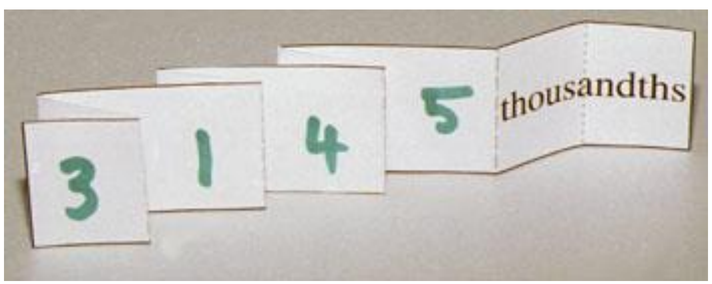
Why decimals?

- Practical Importance
  - Common use in everyday life e.g. economic measures
  - Links to metric measuring
  - Is a statistics *p* value over 0.05?
- Fundamental role of number in mathematics (e.g. understanding 0)
- Known to be complex with poor learning
- A case study of students' growth of understanding, which was able to start from a good research base

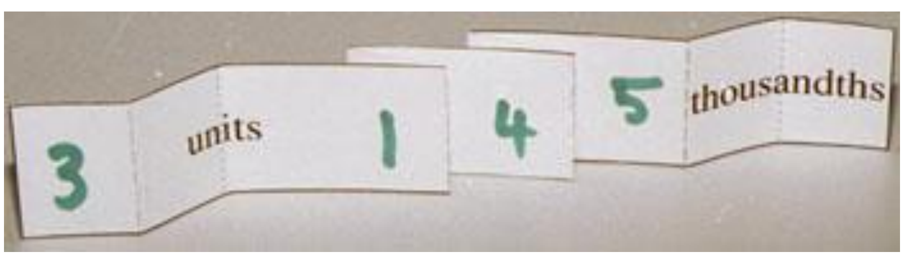
Understanding decimal notation

Why is such a simple rule as rounding hard to remember?





3.145



Similar items – different success

- Select the largest number from
  0.625, 0.5, 0.375, 0.25, 0.125
  Correct: 61%
- Select the smallest number from
  0.625, 0.5, 0.375, 0.25, 0.125
  Correct: 37%
- Why such a large difference?

Foxman *et al* (1985) Results of large scale "APU" monitoring UK.

All sets given here as largest to smallest; not as presented.

Common patterns in answers

0.625 0.5 0.375 0.25 0.125

	Largest	Smallest	
■	0.625	0.125	correct (A)
■	0.625	0.5	well known error "longer-is-larger" (L)
■	0.5	0.625	**identified 1980s "shorter-is-larger" (S)**

Persistent patterns

Select the smallest number from

0.625, 0.5, 0.375, 0.25, 0.125

Option	TIMMS-R International	TIMMS-R Australia
0.125	46%	58%
0.25	4%	4%
0.375	2%	1%
0.5	24%	15%
0.625	24%	22%

Grade 8 – 13 year olds

Longitudinal data from:

- 3204 students
- 9862 short tests
- tests averaged 8.3 months apart
- individuals completed from 1 – 7 tests
- over 3.5 years
- Grades 4 – 10
  - Grades 0 – 6 (ages 5 – 12) “primary”
  - Grades 7 – 12 (ages 12 – 18) “secondary”
- 12 volunteer schools, known to be “typical”
- No special teaching for this sample

Sample data

- Maximum 2 tests per year
- Every test result is a code that signifies students’ interpretation of decimals

ID	Grade 4		Grade 5		Grade 6		Grade 7		Grade 8		Grade 9		Grade 10	
210403026		L1		A1	S3	S5	S1							
300704112							L1	L4	L4	L2	L1			
310401041		L2		L1	U1	U1	L4	U1	U1					
390704012							L1	A1	U1	A1	S3			
400704005							A1	A2	A1	A2	A1			
410401088		L1	L1		L4	L1	L2	A1	A1					
500703003								S1	S5		S3	S3	U1	
500703030								S3	S5		S1	A2		
600703029								A1		U1	A1	A1	A3	

Decimal Comparison Test (DCT2)

We now have better version

Name:______ *Caitlin*______ Class:___ *4H* Date:__ *17 July*_

For each pair of decimal numbers circle the one which is LARGER.

0.4	**0.457**	1.06	**1.053**	**4.4502**	4.45
0.86	1.3	**4.08**	4.7	**17.353**	17.35
0.3	**0.4**	3.72	**3.073**	8.245	**8.24563**
1.85	1.84	2.621	**2.0687986**	**3.2618**	3.26
3.71	**3.76**	**8.052573**	8.514	3.741	**3.746**
4.8	**4.63**	5.62	**5.736**	0.35	**0.42**
0.5	**0.36**	0.5	**0.75**	2.186	**2.954**
0.75	0.8	**0.426**	0.3	**0.872**	0.813
0.37	**0.216**	2.516	**2.8325**	**0.038**	0.04
3.92	**3.4813**	**7.942**	7.63	0.006	**0.53**

Developed as part of the 'Teaching and Learning about Decimal Numbers' project.

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Set of very similar items

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Complex test, easy to complete

- Several items of each type
- Codes require consistent responses
- Items within types VERY carefully matched

Comparison Item		A1	A2	L1	L2	S1	S3	U2
4.8	4.63	√	√	×	×	√	√	×
5.736	5.62	√	√	√	√	×	×	×
4.7	4.08	√	√	×	√	√	√	×
4.4502	4.45	√	×	√	√	×	×	×
0.4	0.3	√	√	√	√	√	×	×

S behaviour: “shorter-is-larger”

- 5.736 < 5.62 (because 736 larger than 62; like fractions or negative numbers)
- 5.736 < 5.62 (62 is “tenths” and 736 is “only thousandths”)

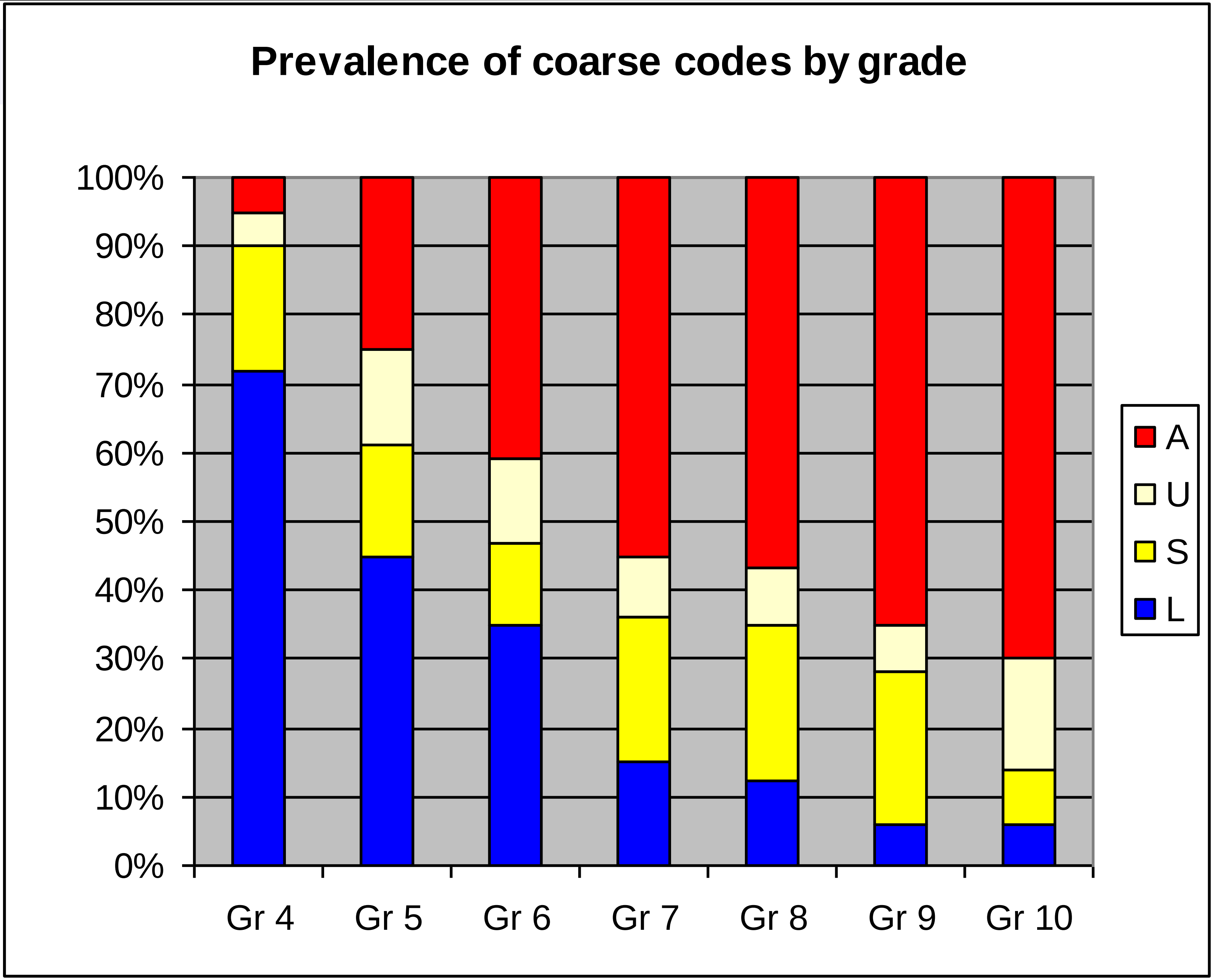
Several reasons for each group of behaviours

S behaviour: false number line analogies with place value

Th	H	T	U	t	h	th
-3	-2	-1	0	1	2	3

- Further right means smaller
- 5.736 “further right” than 5.62
- 0. 6 < 0   because 0 is in “units column”

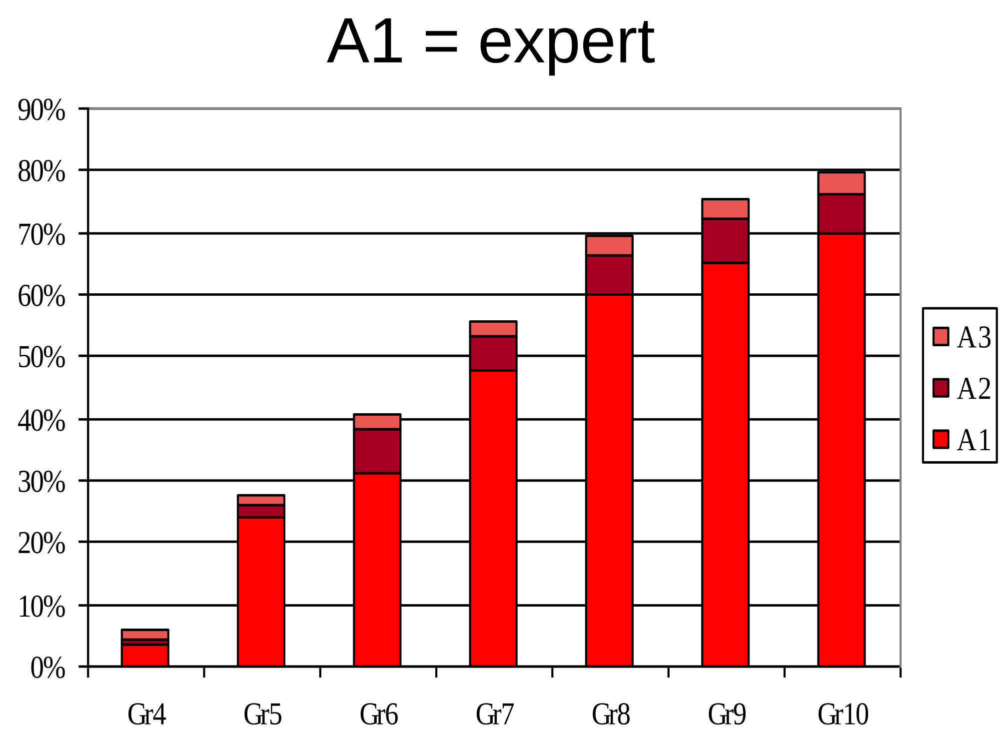
Where are the students in each grade?



Prevalence of coarse codes by grade
100%
90%
80%
70%
60%
50%
40%
30%
20%
10%
0%
Gr 4
Gr 5
Gr 6
Gr 7
Gr 8
Gr 9
Gr 10
A
U
S
L

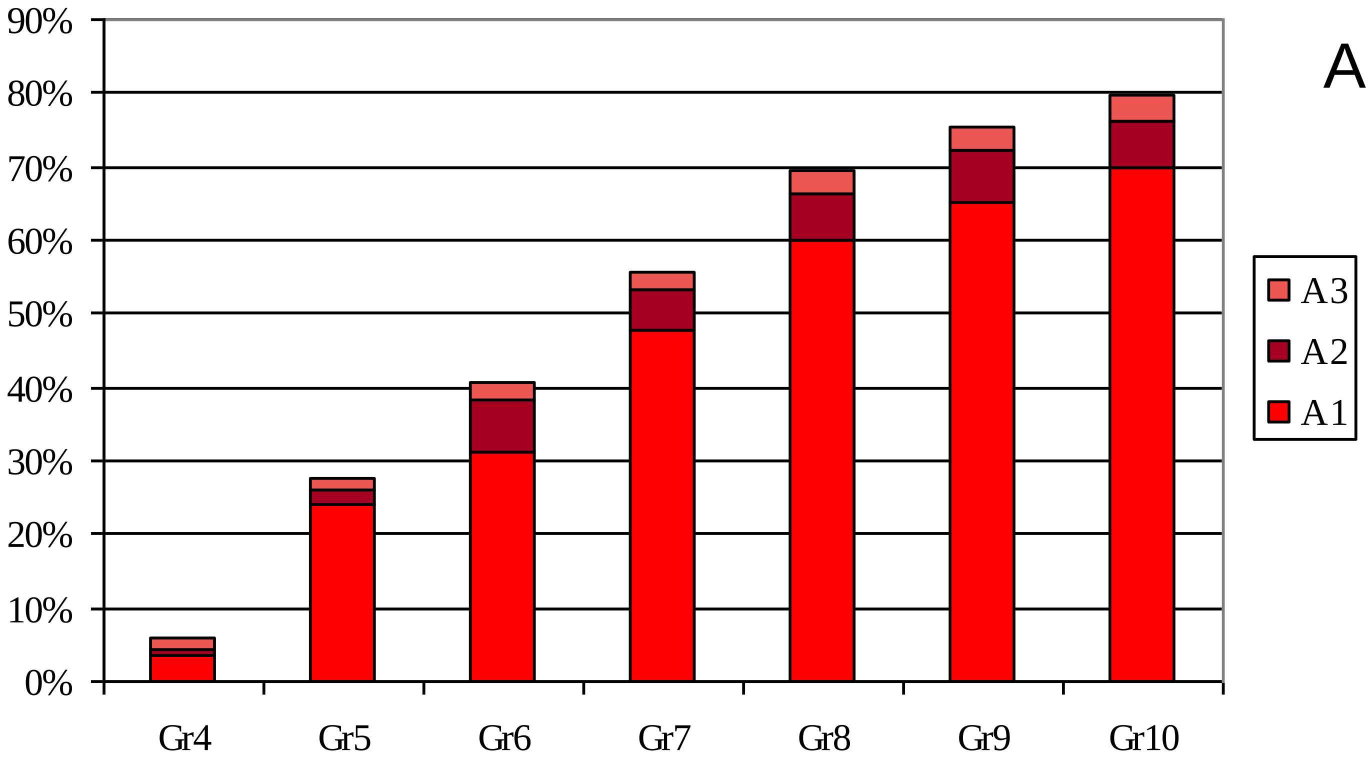
Prevalence of A codes by grade

- 25% expert at Grade 5, 50% in next 4 years, 25% never
- Important in adult education e.g. nurses "death by decimal"
- We know our test overestimates expertise!



Comparison Item		A1	A2	A3
4.8	4.63	√	√	√
5.736	5.62	√	√	√
4.7	4.08	√	√	-
4.4502	4.45	√	×	-
0.4	0.3	√	√	-

Prevalence of A codes by grade



A1 = expert

Note:

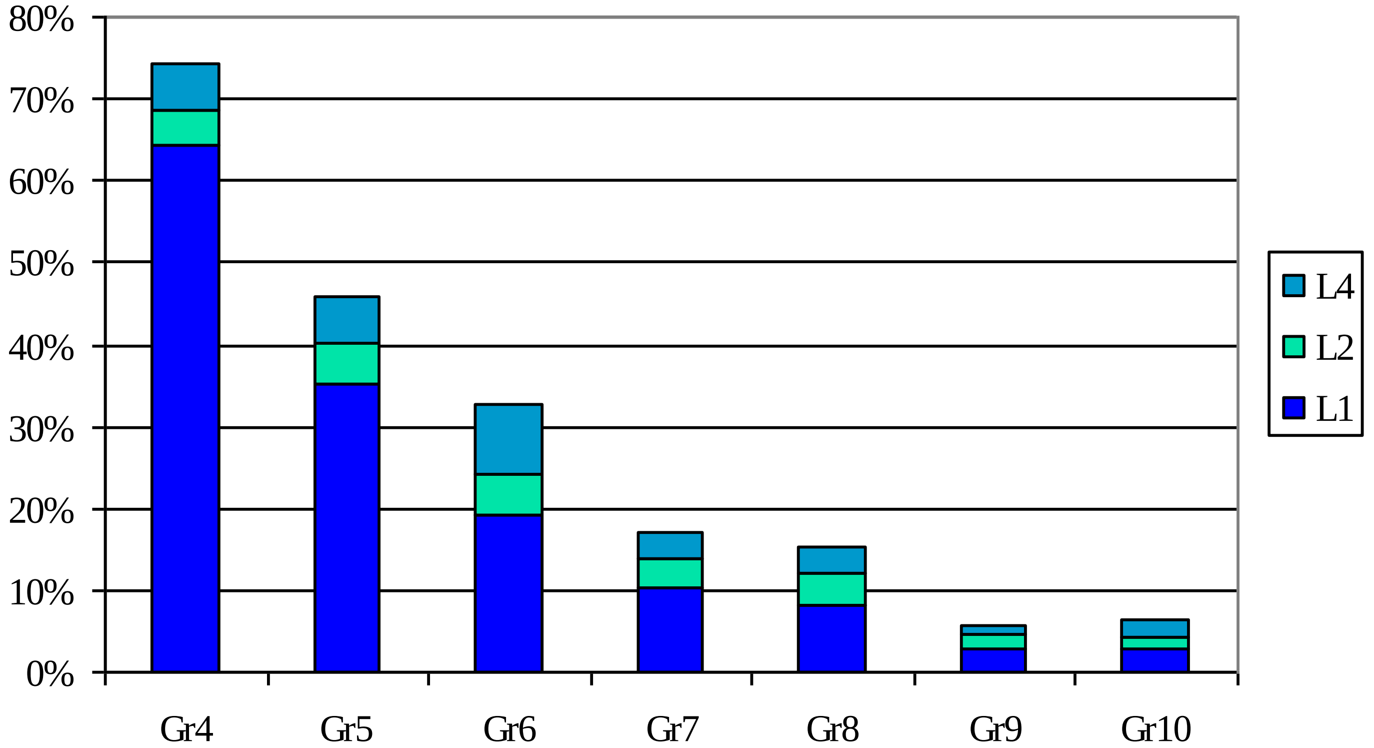
10% in the non-expert A category

These students mostly think that 4.4502 < 4.45 etc

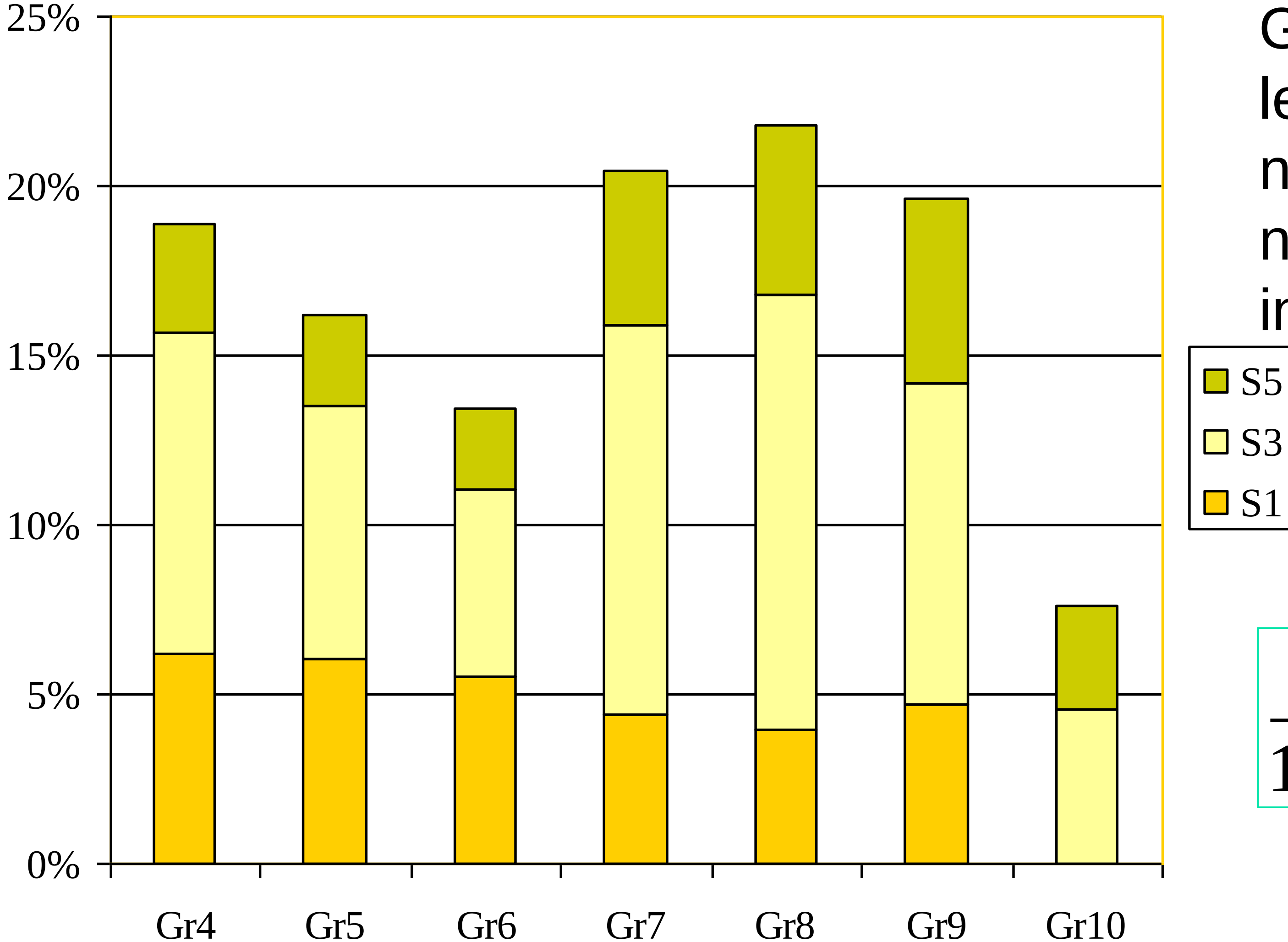
- can usually deal well with decimals to 2 places

Prevalence of L codes

- L drops exponentially (L = 440exp(-0.45*grade))
- L2 about 5% in Grades 5-8: some just accumulating facts, not changing concepts



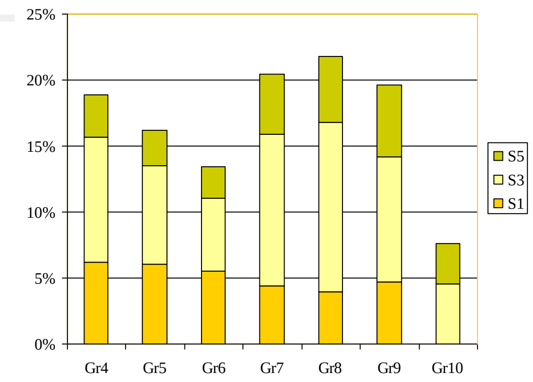
Prevalence of S codes



- Always about 15%
- Maximum in Grade 8 due to learning negative numbers and indices

$$\frac{1}{1000} = 10^{-3}$$

Thinking in S



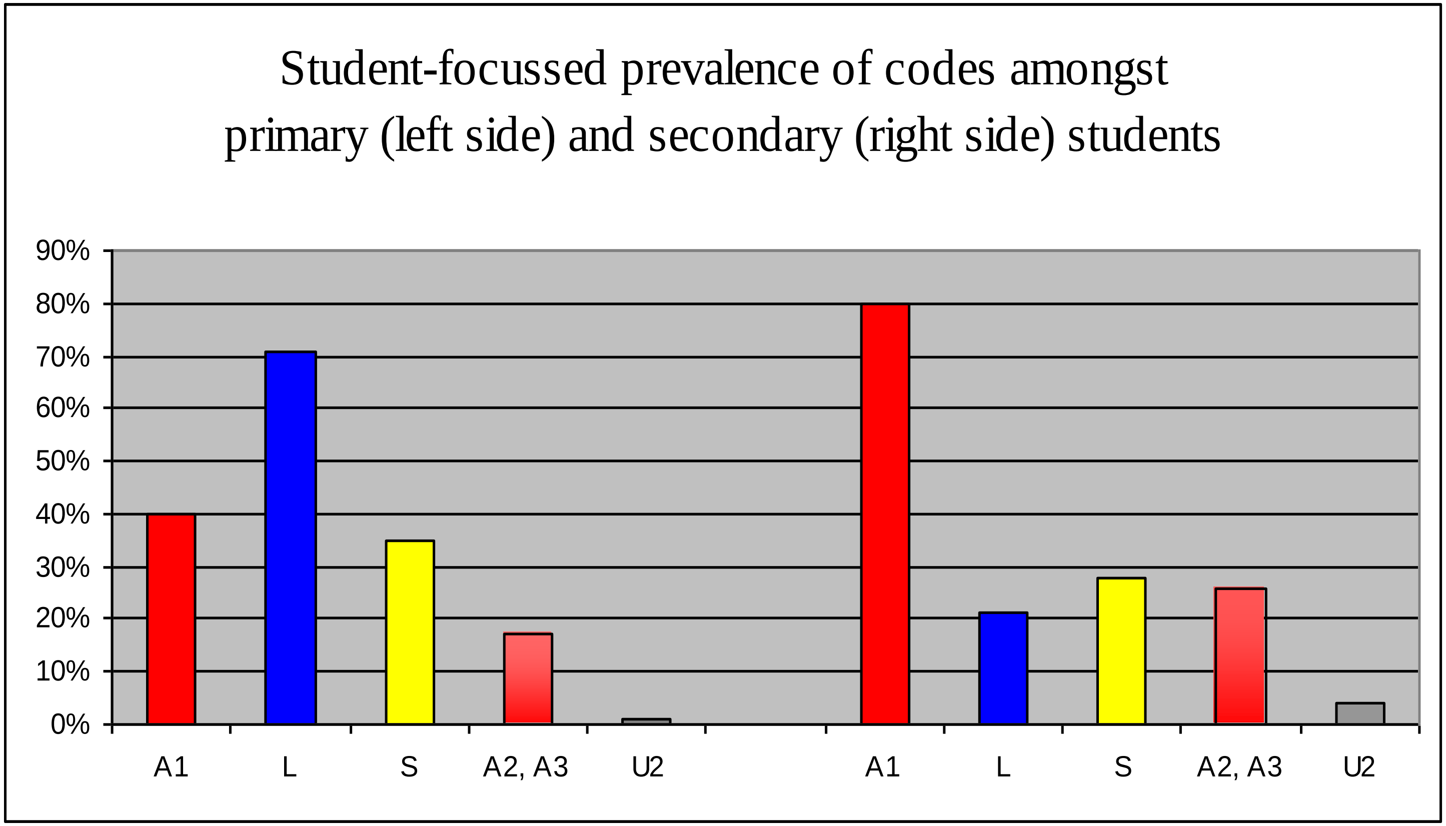
- Around 5% in S1 in all grades
  - 0.6 < 0.7 √
  - 0.5 <0.125 x
  - Immature place value thinking (e.g. “hundredths smaller than tenths”)

- Around 10% in S3 in all grades (more Grade 8)
  - 0.6 < 0.7 x
  - 0.5 <0.125 x
  - analogy with fractions (one sixth, one seventh)
  - analogy with negative numbers (-6, -7)
  - do not think about place value at all

Another look at prevalence

How many students have each thinking at some stage?

Student-focussed prevalence



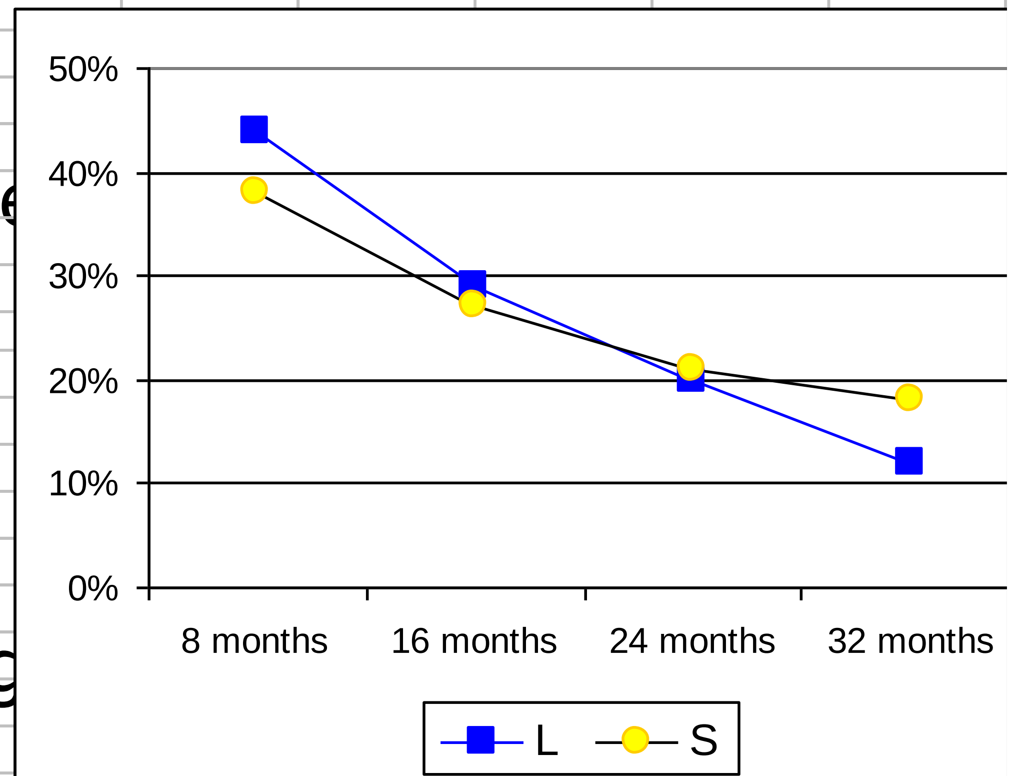
- At some stage during Grades 7 – 10:
  80% of students are expert (A1)
  20% - 30% are in each of the non-expert codes
- Different prevalence for younger students

Persistence
Where do students stay the longest?

- Answer: A1 (expert) - good!

Probability of getting the same test result after 1, 2, 3, 4 tests

- About 35% of L,S,U retest same after one semester
- About 15% retest same after 2.5 years
- Schooling is often not impacting on ideas!
- Our teaching experiments show a little targetted teaching works!



Which students are most likely to become expert on the next test?

- Answer:
- (i) non-expert A (but we think this is false result – they may have little understanding)
- (ii) unclassified – it is better not to have a definite misconception – more open to new ideas?

Lessons about learning

- Different misconceptions have different causes and are impacted differently by the learning environment
  - L1 - naïve, first guess without teaching, decreases in prevalence
  - S – supported by features in the curriculum, operating at deep psychological level, so reinforced especially in secondary school

Need to learn principles, not accumulate facts

- Contrast between orientation to learning principles vs accumulating facts
  - expert: a few math'l principles requiring mastery of a web of complex relations between them
  - For some students and teachers: a large number of facts to learn with weak links between them
- Important not to teach isolated facts

Thank you

Teaching and Learning About Decimals Website

http://extranet.edfac.unimelb.edu.au/DSME/decimals