



A global picture -

'Mathematics education and tools' policy, international trends - French focus -

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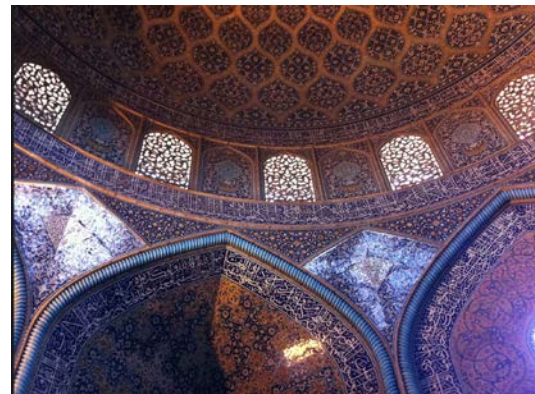


Lecture

February 2016, Farhangian University and Isfahan Mathematics House



Thanks



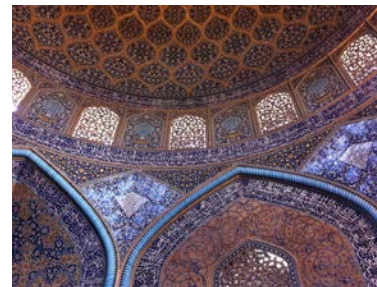
Thank you for inviting me, thanks to the Farhangian University and to the Isfahan House of Mathematics, and to all people helping me during all my stay: colleagues, translator, driver, managers, people in the streets ... I have to say that I had never been so warmly welcomed somewhere

I am honoured and happy to be here, at least for two reasons:

- A new political period seems to take place between our countries, allowing scientific and human relationships to be developed ;
- Iran is one of the place where the true beginning of the history of tools and mathematics took place... A land of culture and science. The Isfahan house of mathematics is here for witnessing this fact !

Since one week...

So many things happened...





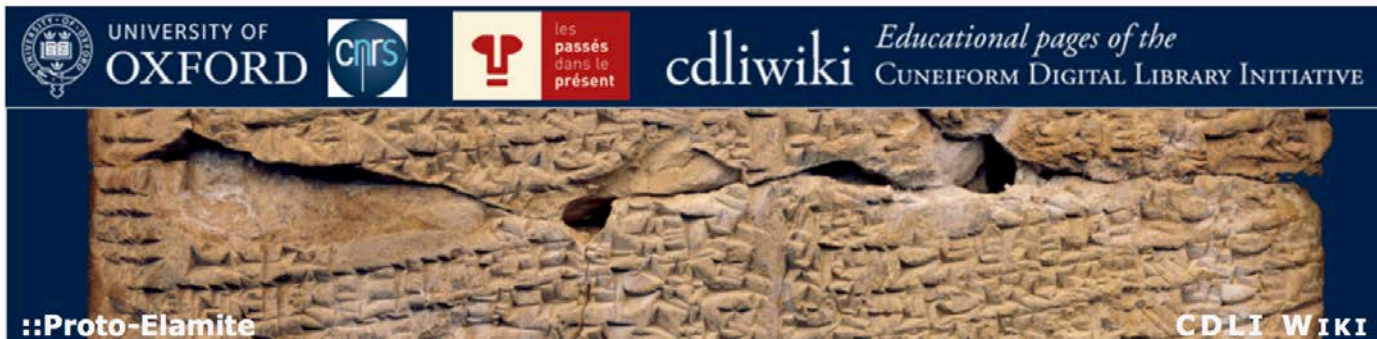
Outlines

- Mathematics and tools, two interrelated stories, an historical point of view
- Mathematics education, a privileged position in the field of research in education, a geographical point of view
- International trends for mathematics education and tools
- The French case



Mathematics and tools, two interrelated stories

Iran, one of the places where things happened



Proto-Elamite is the last un-deciphered writing system from the Ancient Near East (3000 BC in what is today Iran) with a substantial number of sources (more than 1600 published texts).

The majority of the texts are in the Louvre Museum, Paris, and the National Iranian Museum, Tehran



Mathematics and tools, two interrelated stories

Iran, one of the places where things happened



Susa, 3300 BC

Complex tokens representing one sheep, one jar of oil, one ingot of metal, one garment, one honeycomb...

(Schmandt-Besserat, 2009)



Susa, 3000 BC

A period of abstraction, linked to the emergence of writing; combination of various artefacts for counting

(Trouche, 2016)



Mathematics and tools, two interrelated stories

Computations in Mesopotomian Scribal Schools



The shape of the signs, sketched with a pointed calame, is obviously very close to the shape of the vertical and oblique wedge characteristics of the cuneiform writing.

1	2	3	4	5
10	20	30	40	50



Mathematics and tools, two interrelated stories

Computations in Mesopotomian Scribal Schools

Concerning Mesopotamian scribal schools, the situation is exceptionally favourable, due to the huge quantity of school tablets handed down to us. No other educational system of the distant past is as well documented as that of Mesopotamia.

This situation is due to the material used for building the tablets: the clay, a nearly indestructible material.

It also ensues from the reuse of dry and waste tablets as construction material. Trapped in walls, floors or foundations of houses, tablets produced by students and subsequently discarded have escaped other forms of destruction

4000 years after, clay tablets are still alive, speaking to whom is able to understand them. . .

Mathematics and tools, two interrelated stories

Computations in Mesopotamian Scribal Schools

A combination of abstract and concrete computation, needing the use of clay tablets of different natures (metrological tables, reciprocal tables, square roots tables...)



Tablet UM 29-15-192 (Neugebauer & Sachs 1984)



Hand copy made by Proust, personal communication

UM 29-15-192 -Transcription

[2]0
20
6.40
2 šu-si ib ₂ -si _g

a-ša ₃ -bi en-nam

a-ša ₃ -bi igi-
3-gal ₂ še-kam
=====

Translation

$$20 \times 20 = 6.40$$

2 šu-si the side of the square

What is its area?

Its surface is 1/3 še

[a šu-si (= a finger) is a length measuring unit
a še (= a grain) is an area measuring unit]

Interpretation

2 šu-si → 20
20 x 20 = 6.40
6.40 → 1/3 še

Mathematics and tools, two interrelated stories

Computations in Mesopotomian Scribal Schools



$$7.35 \times 7.35 = 57.30.25$$

The resources of the *masters* might have included a complex system of written texts, memorised texts, calculation devices, but only the written artefacts reached us. We have then to reconstruct a rich environment from truncated evidence.

This reconstruction can rely, among others, on this argument: the necessity of artefacts outside of the tablets for doing intermediate computations (Proust 2012)



Mathematics and tools, two interrelated stories

Computations in Mesopotomian Scribal Schools

MesoCalc

A Mesopotamian Calculator

file:///Users/ltrouche/Library/Containers/com.apple.mail/Data/Library/Mail%20Downloads/0D064678-9944-413E-A8C8-32B2EB80C372/mesocalc-2016-01-06.html

B. Mèlès (CNRS, Archives Henri-Poincaré, Université de Lorraine), et
C. Proust (CNRS, SPHERE, Université Paris-Diderot)

$$7.35 \times 7.35 = 57.30.25$$



Mathematics and tools, two interrelated stories

Graphic reason (2D dimension) vs. digital reason (3D dimension)

The development of scribal schools in the late third millennium in Mesopotamia corresponds to a switch in the medium used for the accumulation and transmission of knowledge, from *memorisation*, to *writing*.

This switch could be compared to another major one that of the translation from paper to digital era. Bachimont (2010) opposes the 'graphic reason' (linked to the writing era) to the 'digital reason' (linked to the digital era). The digital reason allows the gathering in the same space of heterogeneous contents, and a multidimensional writing and reading (thanks to hyperlinks).

The supports of knowledge are not only the consequence, but also the cause of knowledge.

Mathematics and tools, two interrelated stories

History of mathematics, history of tools

No real choice for the teachers.

Evolving technology, evolving mathematics, evolving teaching... evolving teachers, seen as Life Long Learners (see the Unesco chair hosted by Farhangian University)

A point of view which will be illustrated during the workshops





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- The French case



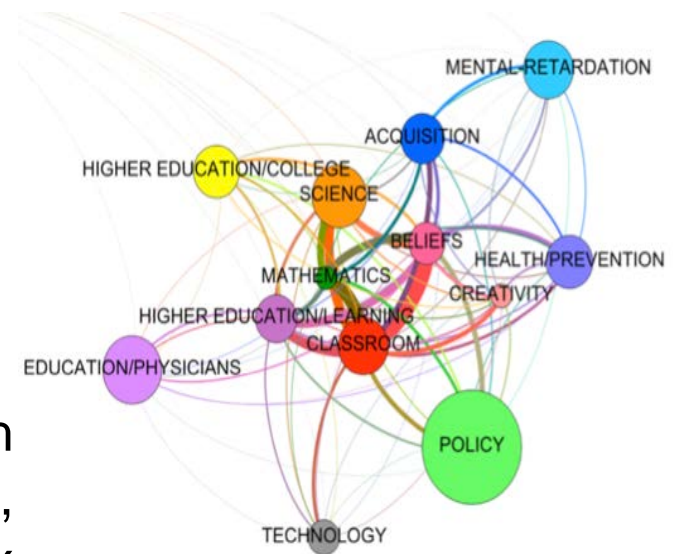
Mathematics education in the field of research in education

Development of mathematics, development of writing, development of learning, interrelated stories.

What is today the place of mathematics education in the field of educational research?

EducMap, a project (2014) of the French Institute of Education (ENS de Lyon), associated with the Institute of Complex Systems

Identifying communities through what they produce: articles (Grauwin & Jansen 2011)





Mathematics education in the field of research in education

The EducMap project

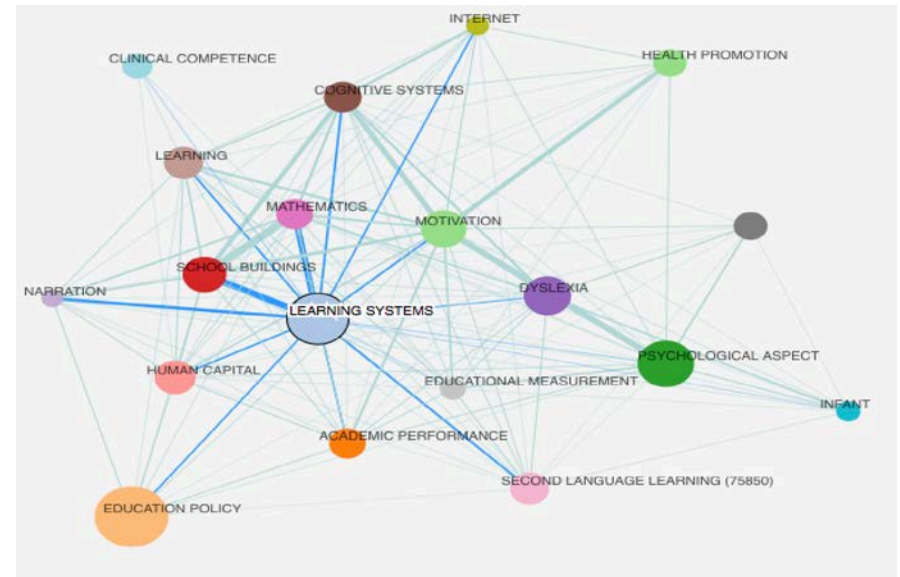
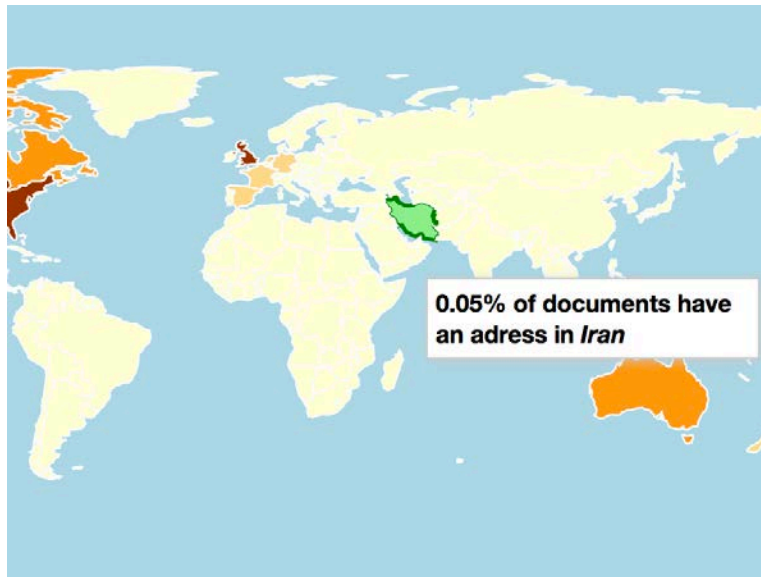
The EducMap project aims at a better understanding of the field of *research in education* thanks to the exploration of bibliographic data. It analyses a given corpus of documents by creating a "Bibliographic Coupling" network, linking articles that share references. An algorithm then groups together dense regions of this network into groups and subgroups that can be interpreted as topical clusters.

Bibliographic data were extracted from the data base SCOPUS. Compared to the Web of Science (its most direct competitor), Scopus is often described as including more journals and sources in the last decades, especially in Humanities and Social Sciences.

However its coverage of the Social Sciences is not perfect and that some biases, such as an under-representation of non-English publications, exist (Lund *et al.* 2016)

Mathematics education in the field of research in education

Visiting the educational research territory



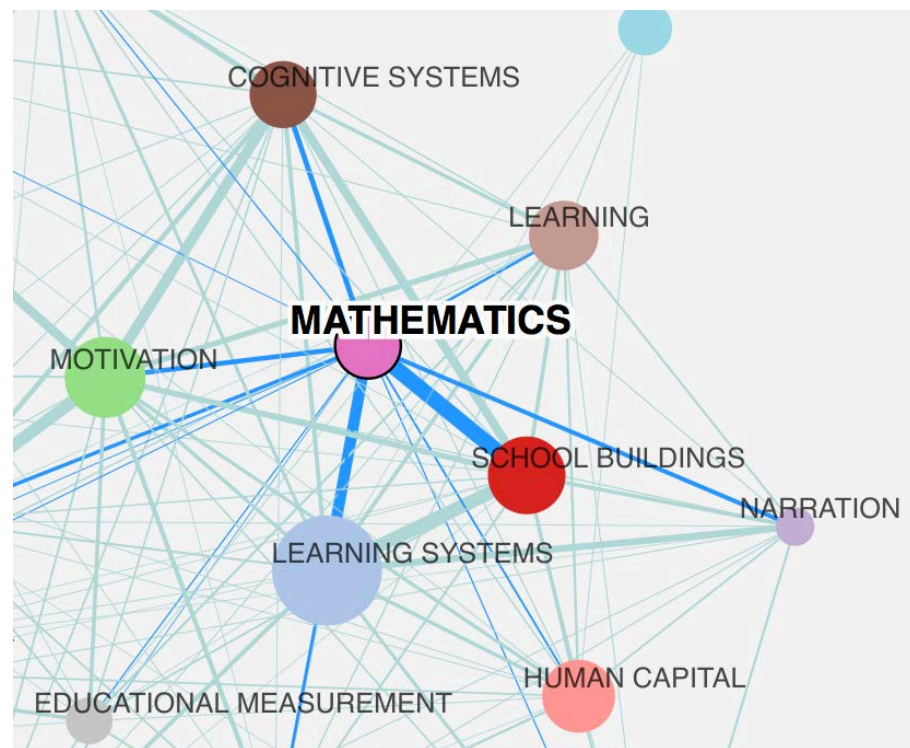


Mathematics education in the field of research in education

Mathematics education, a critical place

The maps reveal the importance of mathematics for studying fundamental questions regarding learning and teaching (computing, proving, representing, critical thinking, problem solving, etc.).

Critical links with other critical communities (learning, motivation, cognitive systems, teachers, curriculum, policies...).

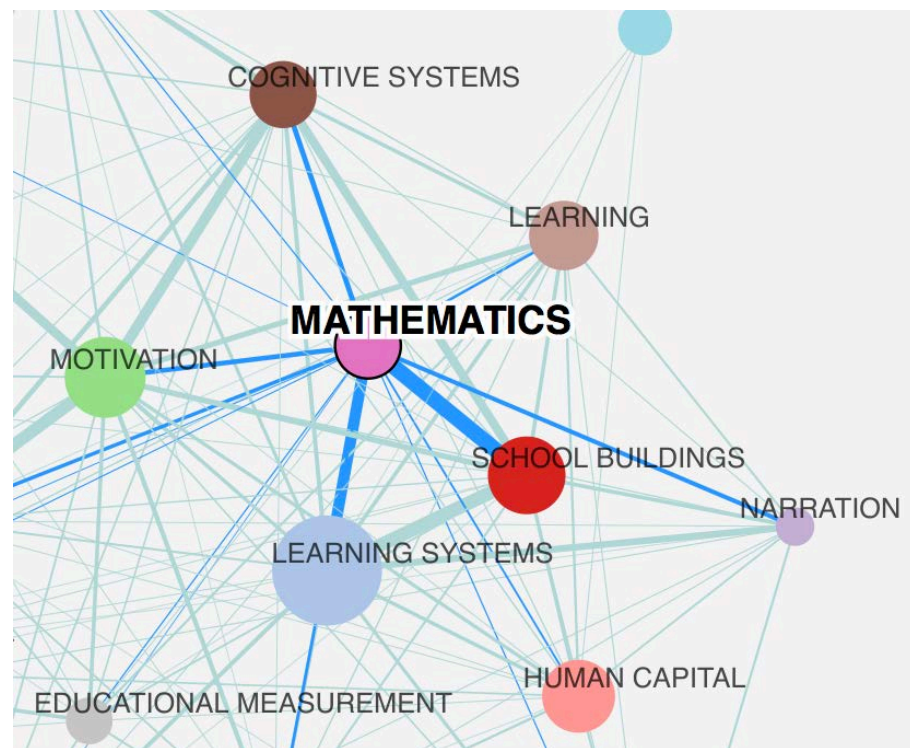




Mathematics education in the field of research in education

As a concrete result...

For educational institutions:
research in math education, a
good investment!





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- **International trends for mathematics education and tools**
- **The French case**



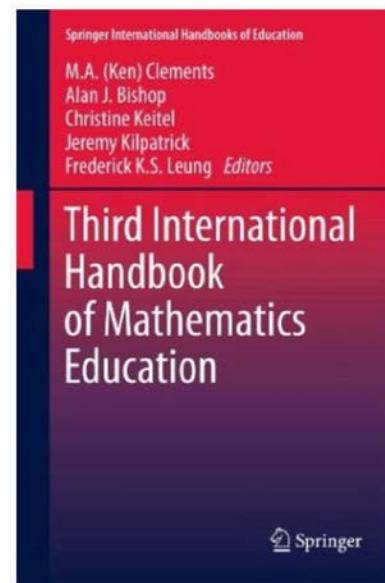
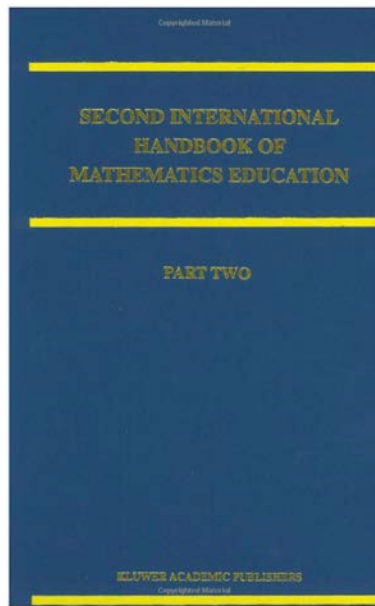
International trends

Looking at trends throughout Handbooks

Handbook of Mathematics Education

2003 - 2013

Two successive handbooks with the same editors A.J. Bishop, M.A. Clements, C. Keitel, J. Kilpatrick, & F.K.S. Leung



International trends

Handbook of Mathematics Education, 2003

Lagrange, J.-B., Artigue, M., Laborde, C., & Trouche, L. (2003),
Technology and Mathematics Education: a Multidimensional Study of
the Evolution of Research and Innovation

A paper based on a large survey of the literature of
research (1994-1998, 800 papers)

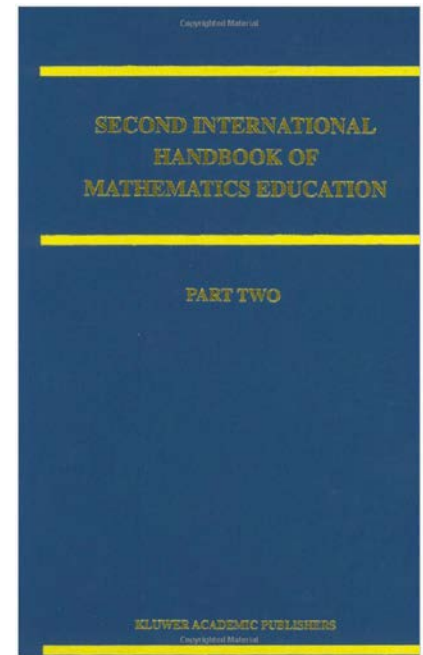
End of a naive period (« spontaneous positive
effect of tools »), beginning of scaling-up
reflections

Integrating technology takes time

Integrating a technology needs to take into
account its potential and constraints

Integrating technology needs to design specific
situations (problems)

The critical role of the teacher



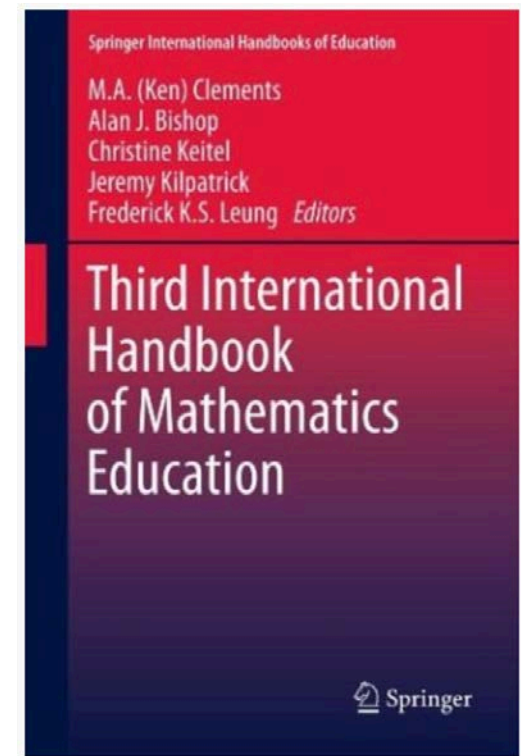
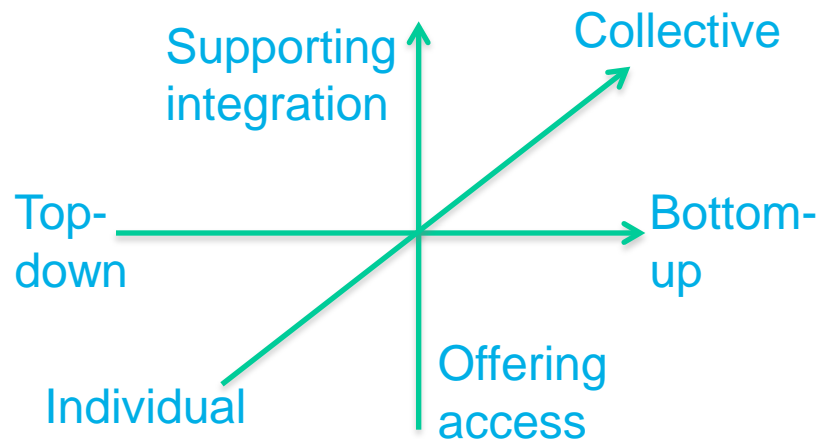
International trends

Handbook of Mathematics Education, 2013

Trouche, L., Drijvers, P., Gueudet, G., & Sacristan, A.I. (2013),
Technology-Driven Developments and Policy Implications for
Mathematics Education

From the study of the educational
policies, a double emergence:

- From technology to resources
- From individual to collective



The typical example of Geogebra tube



The typical example of Sésamath in France

Collectives gathering for collaboratively designing resources: Sesamath (5000 teachers, 100 working groups, designing e-textbooks and exercises used by 100000 teachers...). A platform including a laboratory for steering collaboration

Announcing a new period for free e-textbooks? Teachers as designers of their own resources? Teachers Life Long Learners?



Mathematics for everybody

Working together, supporting one another, communicating

The typical example of MOOC

Institutional initiatives: Massive Open Online Courses (MOOC)

A French MOOC for “teaching math with ICT”

A pedagogical team of 10 teacher trainers, 3000 trainees, considered as active participants

Development of collaborative projects on the basis of small teams, cross assessment

New edition beginning March 8th (in French, but open to Iranian colleagues!)



International trends

A renewing of the field of research on math textbooks

Textbooks as a part of larger field of teachers' resources



TSG 38 **Research on resources** **(textbooks, learning materials etc.)**

Co-chairs:

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Team members:

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Sebastian Rezat (Germany)
Jana Visnovska (Australia)



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Curriculum reform in France, main steps

2012 - The higher council for teaching programs (CSP)

CONSEIL SUPÉRIEUR
DES PROGRAMMES



Created by the 2012 law for re-grounding the republican school

Composed of scholars, experts of educational issues, representative of the nation and of the society

Its mission: formulating answers to institutional questioning, or to self questioning

Michel Lussault, director of the French Institute of Education, is the current president of this council

Curriculum reform in France, main steps

2014 – A charter for the teaching programs

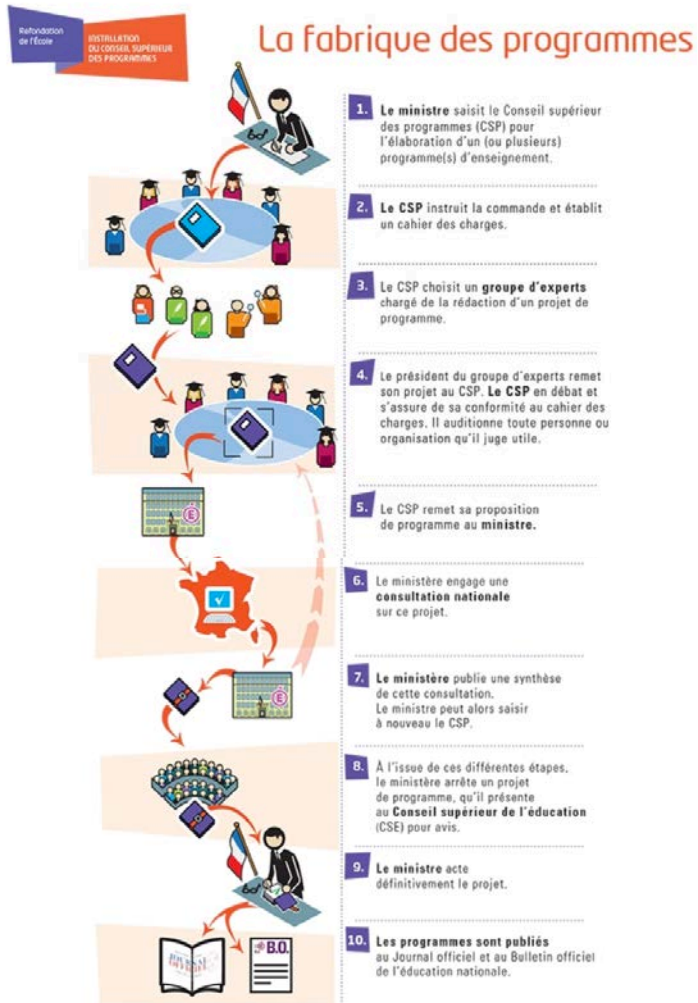


Under the responsibility of the CSP
Defining the true nature of a teaching program, as a living entity, its conditions of birth, and of change

Defining the relationships between a teaching program and the whole society: the common core of knowledge and competencies

Steering the interactions between the teaching programs, the educational research and teachers training.

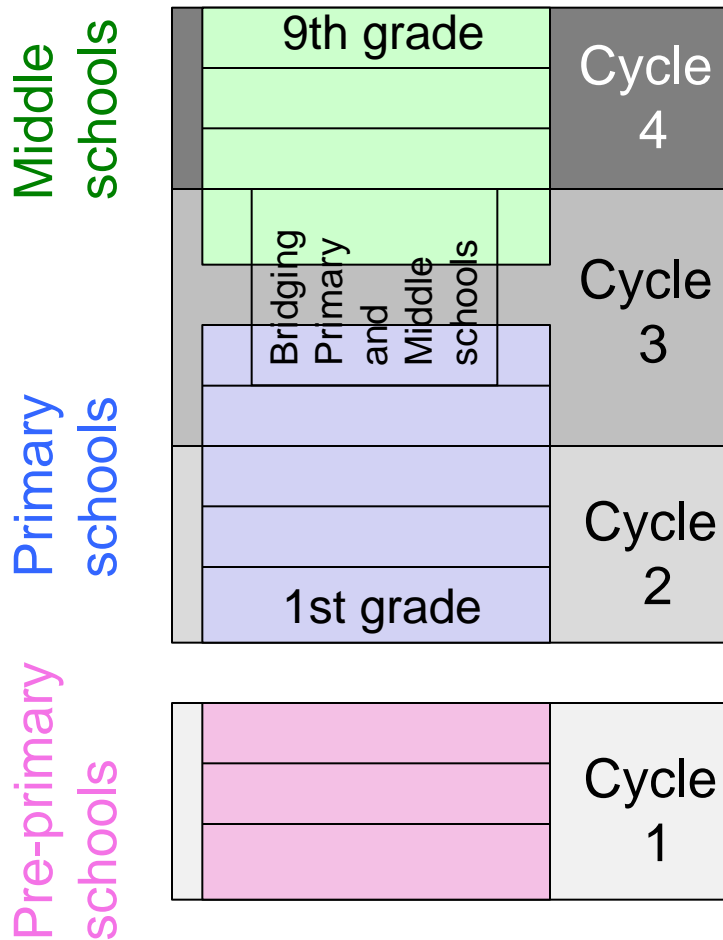
Curriculum reform in France, main steps 2015 – Defining the “programs factory”



- The Ministry asked the CSP to produce a new curriculum
- The CSP appointed a group of experts who writes a first version and submits their proposal to the CSP
- After discussion, the CSP proposed its project to the Ministry, who launches a national consultation
- After analyzing its results, the Ministry asked the CSP for a new version

Curriculum reform in France, main steps

2016 - Implementing the new programs 1st – 9th grades



An implementation linked to a global reform of the primary and middle schools

An organisation based on four 3-years cycles: the targeted competencies and knowledge have to be reached at the end of each cycle, leading teachers to work together inside a given cycle

The third cycle having the essential role to establish a continuity between primary and middle schools

Curriculum reform in France, main steps

2016 – The structuring role of the common core

Part 3: specific to each discipline

From the contribution to the common core to the content to be taught and to the context grounding its meaning

Part 2: from common core to disciplines

Contribution of each discipline to the general objective of the cycle. For example, in mathematics : switching from a representation to another one, etc.

Part 1: general objective of the cycle

For examples: historical dimension of knowledge, abstracting and modelling, collaborating, creating...

Thinking the disciplinary contents from the common core

The program of each cycle is structured in three parts of description,

A structure aiming to support the evolution of teachers' work, a curriculum being no more the addition of disciplines, but a *global project*

Curriculum reform in France, main steps

2016 – Freeing teachers to collaborate cross the disciplines

Economical & professional world

Langages & cultures from Antiquity

Langages & cultures from abroad/regions

Culture & artistic creativity

Elements of a school based curriculum, the interdisciplinary practical teaching (EPI) are to be chosen by the school (6 over 8 fixed by the curriculum)

Ecology & sustainable development

Information, communication & citizenship

3 hours a week to be organised by volunteers teachers

Body, health, well being & security

Sciences, technology and society

Focusing on the curriculum in Mathematics

Linking competencies and knowledge

Attendus de fin de cycle

- Utiliser les nombres pour comparer, calculer et résoudre des problèmes
- Comprendre et utiliser les notions de divisibilité et de nombres premiers
- Utiliser le calcul littéral

Knowledge and associated competencies

Exemples de situations, d'activités et de ressources pour l'élève

calculer et résoudre des problèmes

Utiliser diverses représentations d'un même nombre (écriture décimale d'un nombre, écriture fractionnaire, représentation sur une droite numérique, etc.).

Nombres relatifs

- Nombres relatifs opposés.
- Fractions et opérations sur les fractions.
- Les préfixes de nano à giga.

Competency: to switch from a representation to another one

Knowledge: decimal numbers

Rencontrer diverses écritures dans des situations variées (par exemple nombres décimaux dans des situations de vie quotidienne, notation scientifique en physique, nombres relatifs pour mesurer des températures ou des altitudes).

Associer à des objets des ordres de grandeurs (par exemple, la taille d'un atome, d'une bactérie, d'une cellule pulmonaire, la longueur de l'intestin, la capacité de stockage d'un disque dur, la vitesse du son et de la lumière, la population française et mondiale, la distance de la Terre à la Lune et au Soleil, la distance du Soleil à l'étoile la plus proche).

Prendre conscience que certains nombres ne sont pas rationnels.

Focusing on the curriculum in Mathematics

Linking knowledge and situations grounding its meaning

Attendus de fin de cycle

- Utiliser les nombres pour comparer, calculer et résoudre des problèmes
- Comprendre et utiliser les notions de divisibilité et de nombres premiers
- Utiliser le calcul littéral

Knowledge and associated competencies

Utiliser diverses représentations d'un même nombre (écriture décimale ou fractionnaire, notation scientifique, repérage sur une droite graduée) ; passer d'une représentation à une autre.

- Nombres décimaux.
- Nombres d'opposé.
- Fractions, fractions décimales.
- Définition de la racine carrée ; les carrés parfaits entre 1 et 144.
- Les préfixes de nano à giga.

Knowledge
From Nano to Giga

Examples of situations, of activities, of resources for the students

Rencontrer diverses écritures dans des situations variées (par exemple nombres décimaux dans des situations de vie quotidienne, notation scientifique en physique, nombres relatifs pour mesurer des températures ou des altitudes).

Atoms and bacteria
Capacity of hard disks
Speediness of sound and light
Distance between earth and sun

l'étoile la plus proche).
Prendre conscience que certains nombres ne sont pas rationnels.

Focusing on the curriculum in Mathematics

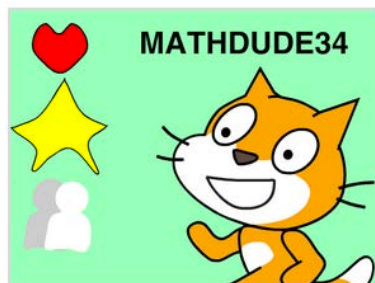
Introducing algorithm in the 4th cycle curriculum

In this cycle, students discover *programming*, through personal and collective projects.

What is at stake: developing new methods, opening new windows on notions of *variable* and *function*, and on *proving*.

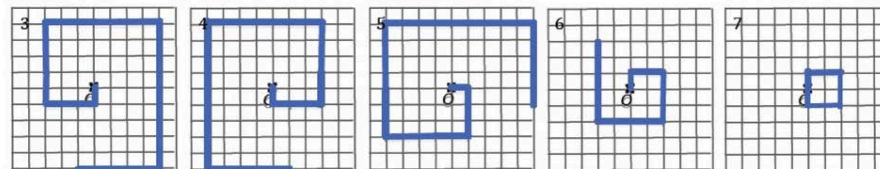
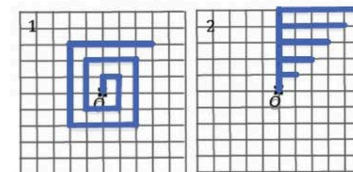
A shared responsibility of the math teacher and of the technology teacher

Needing teacher training and new resources.....



Why I Don't Use Li...
by mathdude34

Se placer en O
Pour i entier de 1 à 5 faire
 Avancer de i cases
 Tourner de 90° vers la droite
 Avancer de i cases
Fin du pour



Scratch, MIT

Institutional resources supporting teachers

Focusing on the curriculum in Mathematics

Introducing algorithm in the 4th cycle curriculum

A beaver

An increasing interest for programming, supported by the actors of the domain (computer scientists, companies..)

A national competition organized in schools from grade 4

A discussion between mathematicians and computer scientists: who has to teach programming?



**CONCOURS
CASTOR INFORMATIQUE**
castor-informatique.fr

14 au 21 novembre 2015

Gratuit et en ligne
Ouvert aux CM1-CM2

18 questions interactives - 45 minutes
En ligne, sur ordinateur ou tablette
Seul ou en binôme
Aucune connaissance préalable requise

Collier de perles

Plus d'énigmes sur :
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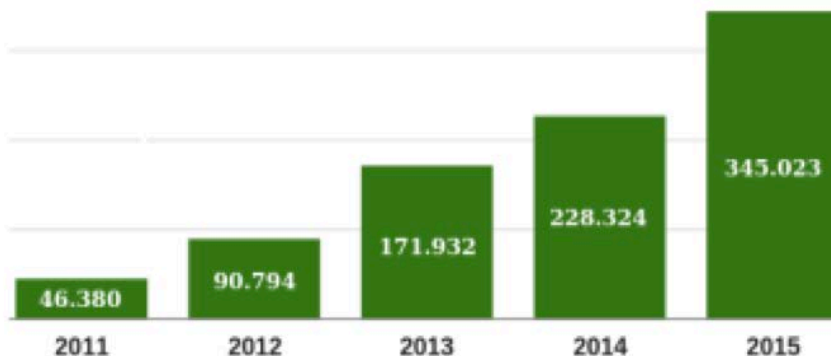
Castor veut fabriquer ce collier.

Quelle séquence de 4 perles, répétée, lui permettra de fabriquer le collier ?

Fort de son succès avec 230 000 participant-e-s en 2014, le Castor Informatique s'ouvre aux élèves de CM1 et CM2. Il offre une initiation ludique à l'informatique inscrite dans la plan numérique pour l'éducation.

ENS IRIA

Contact :
info@castor-informatique.fr



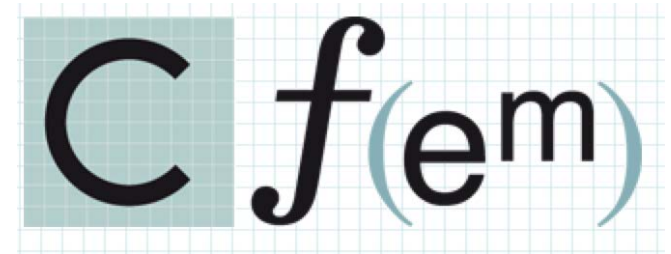
Focusing on the curriculum in Mathematics

A true engagement of the actors in the field, via the CFEM

Mathematics, the discipline which led the most intense discussion with the CSP, through the CFEM, French Commission for Mathematics Teaching

A platform gathering scholars, mathematicians, mathematics teachers, mathematics education researchers, “Académie des sciences”...

CFEM is the French section of the International Commission on Mathematical Instruction (ICMI), created by the International Mathematical Union (IMU) in 1908 in Roma.



Focusing on the curriculum in Mathematics

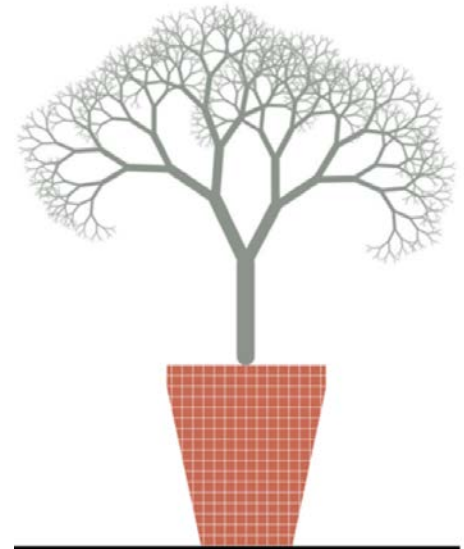
French mathematicians, French educators and society

ICMI, a place where mathematicians and psychologists had been interacting (e.g. Piaget, Freudenthal, Polya...)

In France, an ancient commitment of the mathematics community towards teaching and society (Poincaré, Dieudonné, Choquet,... Villani)

The move for changing mathematics (“Modern Math reform”) and the move for changing society (May 1968 events), lead to the creation in 1970 of the IREM (Institutes for Research on Mathematics Teaching) in each University.

A place where teachers of all levels reflect on their own activity and design innovative resources



From the curriculum to the Mathematics Strategy

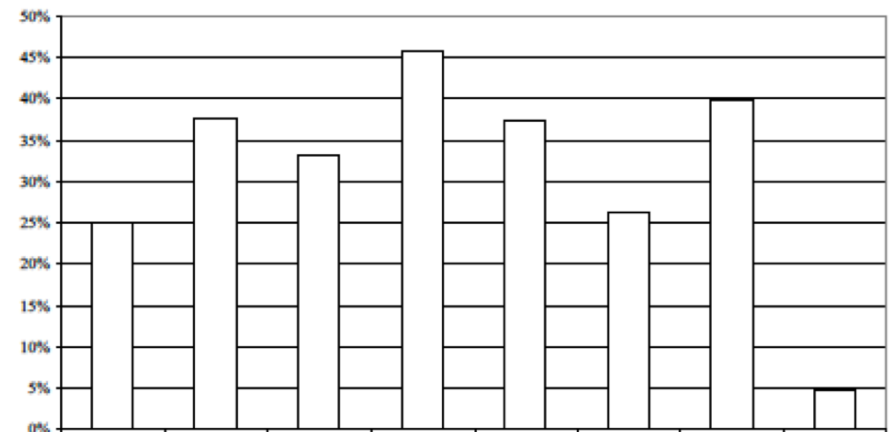
Mathematics teaching facing two major issues...

A first issue in mathematics learning: an increasing gap, from the beginning to the end of the middle school, between “good” and “weak” students in mathematics (OECD 2016)

A worrying result, as the success in mathematics is strongly linked to the language ability, to the mastering of space, to the memory and to the classroom attention

Besides students seem to have a good image of mathematics, as a teaching discipline

France



First
language

Math

Sport

From the curriculum to the Mathematics Strategy

Mathematics teaching facing two major issues...

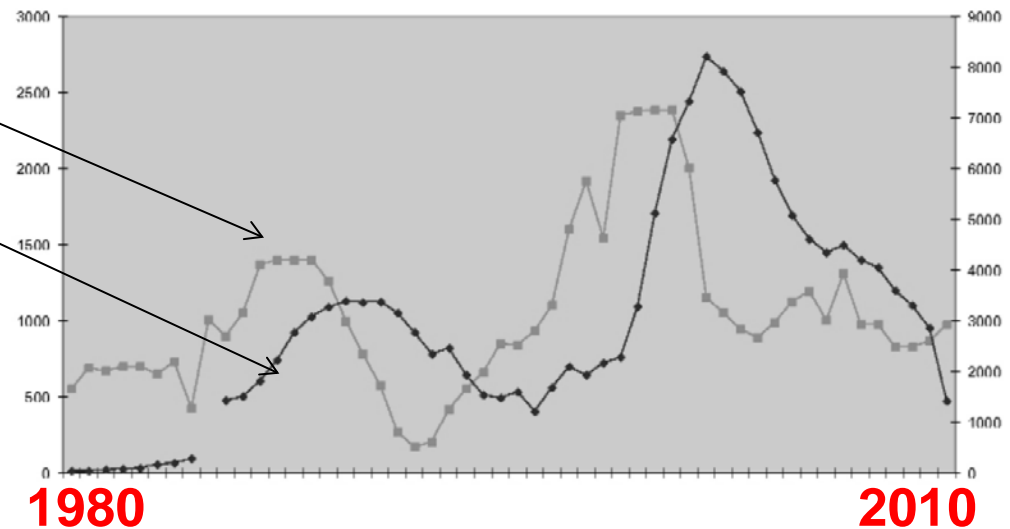
A second issue in mathematics teaching: an increasing deficit of math teachers, due to the lack of applicants to the competitive examination for becoming a math teacher. This year: the Ministry offered 1440 positions, and only 1097 teachers have been recruited

A strong questioning of the attractiveness of mathematics, and of their potential for developing deep learning

Number of positions offered

Number of applicants

The French situation, regarding the recruitment of teachers



From the curriculum to the Mathematics Strategy ... and, besides, a strong potential of mathematics in France

An international acknowledgement of the French school of mathematics (Fields medals to Cédric Villani, Artur Avila...) and didactics of mathematics (Klein and Freudenthal medals to Michèle Artigue and Yves Chevallard)

A huge number of creative initiatives, individual as well as collective, an abundance of digital resources



MAISON DES MATHÉMATIQUES
ET DE L'INFORMATIQUE



From the curriculum to the Mathematics Strategy

Towards a Mathematics Strategy



The awareness of the necessity for rallying the efforts of all the components of the mathematics education community, leads in 2014 to the proposal, by the CFEM, of a *strategic program for supporting mathematics teaching*.

December 2014: the French Ministry announces a Strategy resting on 10 points


The French Ministry of education, Najat Vallaud-Belkacem, between two French mathematicians, Cédric Villani and Nalini Anantharaman, announcing the Math Strategy



From the curriculum to the Mathematics Strategy

Towards a Mathematics Strategy

- ① A new curriculum
- ② Learning mathematics as a deep and active learning
- ③ Taking into account the research in the field
- ④ Improving teacher training
- ⑤ Developing mathematics teaching attractiveness
- ⑥ Encouraging scientific careers
- ⑦ **Promoting digital environments**
- ⑧ Fighting gender stereotypes
- ⑨ Increasing the value of mathematical educative activities, in and out of schools
- ⑩ **Developing a national portal for mathematics teaching resources**



Needing means + political choices (e.g. for teachers recruitment)

To be opened next May

Iran-France, towards new interactions?

A strong common interest for mathematics

A time of reform of curricula

New digital environments

New challenges for teacher training

New means for collaborating at a distance...

Opening a new page for our common history?



۱/۶ ۱/۶ ۱/۶ ۱/۶ ۱/۶ ۱/۶ ۱/۶ ۱/۶ ۱/۶ ۱/۶



Cédric Villani



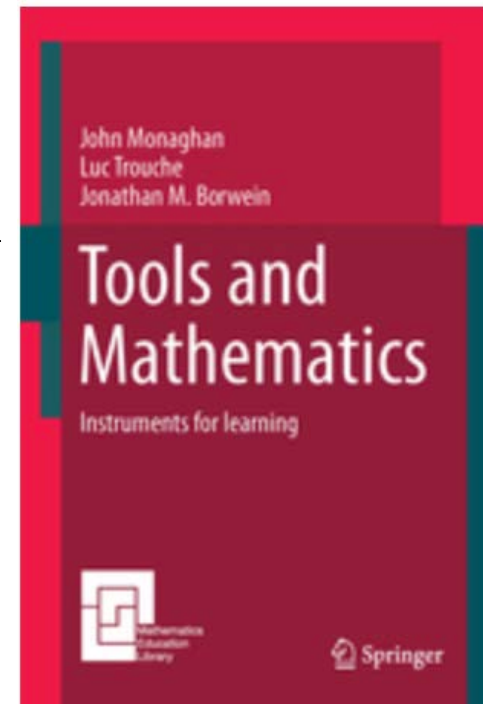
ENS de Lyon / Farhangian University

UNESCO Chair on *Teachers as Lifelong Learners* at Farhangian University UNESCO Chair on Teachers (Prof. Dr. Mehrmohammadi)

UNESCO Chair on *Training teachers for the XXI century* at ENS de Lyon (Prof. Luc Ria)

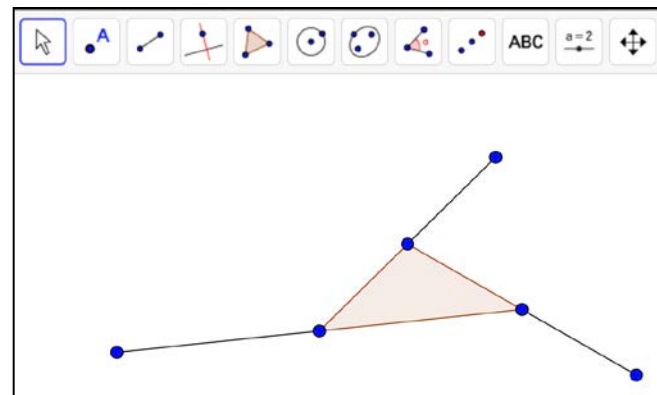
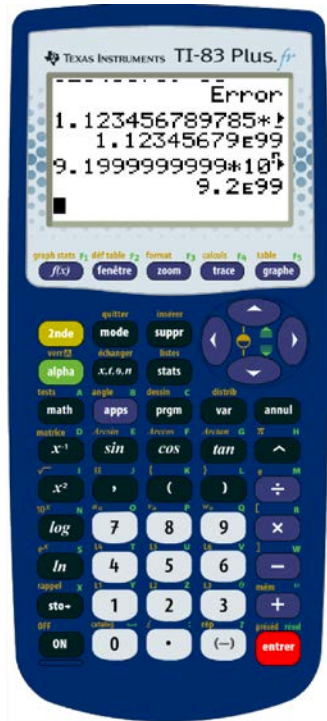
Opportunities for new collaborations...

To be published in one month: mathematics, tools, and learning
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As a follow-up...

Two-days workshop to come!

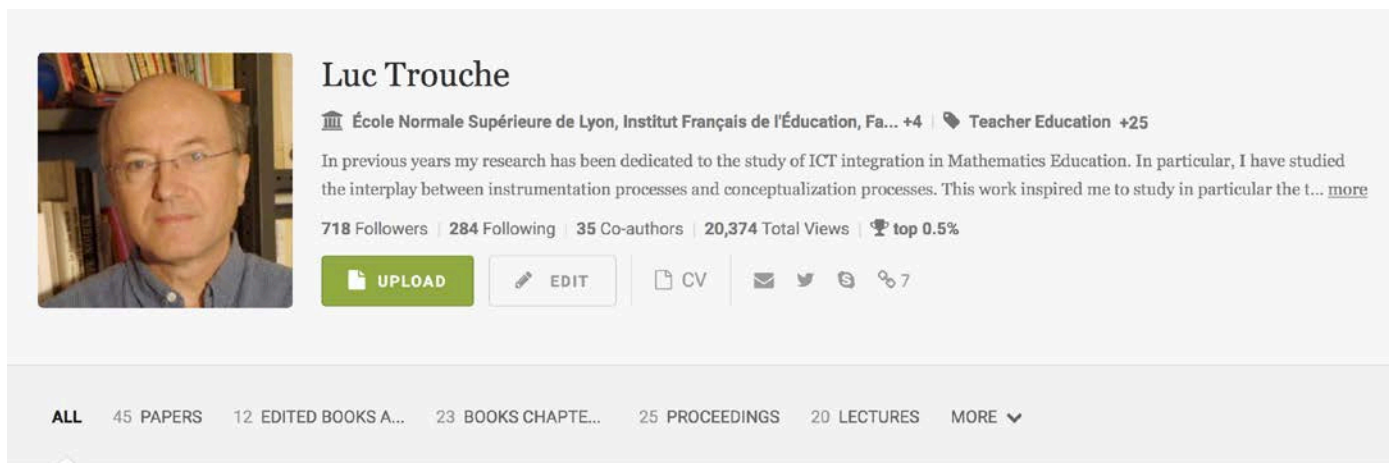


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In previous years my research has been dedicated to the study of ICT integration in Mathematics Education. In particular, I have studied the interplay between instrumentation processes and conceptualization processes. This work inspired me to study in particular the t... [more](#)

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