SPEXXX

Alympiad Finale

March 17-18, 2023

Garderen

Colophon

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INSTRUCTIONS FINALE MATHEMATICS A-LYMPIAD 2023

IN ADVANCE:

- First read the full text of the assignment so that you know what you have to do this weekend.
- Divide the tasks where possible.

PLANNING:

- Keep an eye on the time while working on the different parts.

- You must both hand in anassignment and prepare a presentationMore information about the presentation will follow during the weekend.

- Plan in advance who will start work on what when.

- Saturday afternoon, before lunch, at 12.30 pm you must hand in the USB stick with the assignment (as one document).

HANDING IN:

- Saturday afternoon no later than 12.30 pm: the USB stick with the assignment on it.

  o Hand in the assignment as one document, preferably a PDF.

  o Check on several computers if the PDF is readable.

  o The jury receives digital copies of the submitted file, so you may use colors in the assignment.

  o You are responsible for whether or not the submitted document is readable and processable.

JUDGING:

- Combine all your findings into a clear report that can be read independently (without the assignment). The focus will be on the final assignment with substantiation and arguments.

- When judging the report, attention will be paid to, among other things:

    o the elaboration and reporting of assignments 1 to 3 and the final assignment;
    o the completeness of the answers to the different parts;
    o the use of mathematics;
    o the argumentation used and justification for the choices made;
    o the depth to which the various assignments are answered;
    o how the whole is presented: form, coherence, readability, illustrations, etc.;
    o originality and creativity.

    Have fun and good luck!

Introduction

This Alympiad final assignment is about the game Spexxx. This is a strategy game with dice. You will play the game a number of times to get a grip on the rules and strategies. Of course, you will also do (probability) calculations and mathematical reasoning, and you will take a critical look at the scoring. Finally, you will design a scoring system and perhaps also new rules for a simplified version of this game.

An explanation of the game can be found on YouTube:

NL: https://www.youtube.com/watch?v=7I_wGd7okjw

EN: https://www.yucata.de/en/Rules/Spexxx

Dice

The chance that you can place a block on, for example, a red square can hardly be calculated. But we can approach that probability empirically by rolling the dice and repeating enough times.

Each team will therefore 'generate data'. Inside the envelope you will find instructions on what to throw and what strategy to follow. Submit your results to the organization.

Please note: see at the end for envelope contents instructions.

Spexxx

The only way to get a mathematical grasp of the game is to play it a number of times. In this assignment we only look at the variant for two people. Therefore, play the game in pairs and discuss aloud what considerations you make.

Assignment 1 Calculating(probability) with Spexxx

When playing Spexxx you always throw five dice. Throwing five dice is of course a source for countless probability assignments.

a. Calculate the probability that you roll five of the same number in one throw.

You want to place a block in the purple area of the playing field. You have separated three 5s after two throws and you may throw two more times. On the third roll, you roll a 1 and a 3. [Note: The numbers are in Dice, a "dice font"]

b. Which is smarter: leave the 1 and try to add another 1 on the fourth roll, or roll both dice again?

To interfere with your opponent, you want to land on the blue square $4, 4, 4, 1$. It's not at all easy, but in the third throw you suddenly have:

$1, 3, 4, 4, 6.$

c. Which dice do you leave behind and which do you throw again in the fourth throw? And what are the chances of achieving your goal?

Ed comes up with the following reflection: 'The chance of throwing silver (five successive ones) is actually smaller than the chance of throwing gold (five of the same kind): there are only two possibilities, namely $1, 2, 3, 4, 5$ and $2, 3, 4, 5, 6$. For gold, you have six different options.

d. See if Ed is right.

Assignment2Place strategic blocks

You place your first cube in the purple field on the square you see in the image above.

a. In how many places can you place your next cube so that you can get a score on your third turn (so that you have three cubes in a row)? There is no opponent here.

b. Give a reasoned estimate of the probability that you will indeed succeed in your third turn (without an opponent).

Spexxx is also a fascinating game without dice. In this assignment, you will play with two people and each participant receives twelve blocks. Take turns placing a block on the board and determine the score.

c. What is the maximum number of points you can score with twelve blocks if you are not hindered by an opponent?
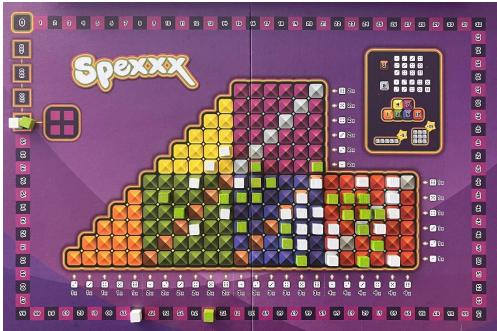
You could also team up to get as many points as possible together.

d. Determine the maximum possible score for you and your opponent together.

At every turn the dilemma arises: do I go for my own gain or do I block my opponent? Would there be an always winning strategy?
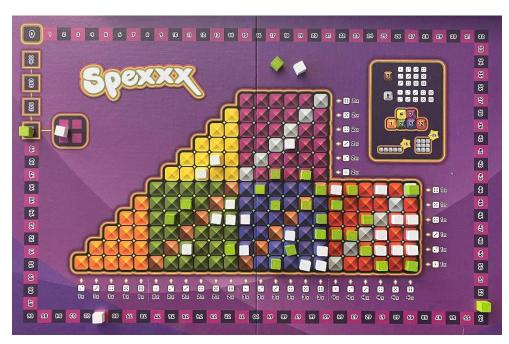
e. Investigate whether there is such a winning strategy and provide a substantiated report of both the search and the strategy found.

Assignment3Game situations

Ruurd (white) and Ruben (green) play this game very often. At one point, this was the endgame. Ruben has one block left to place.

a. Is the score so far correct?

b. Give a substantiated estimate of the chance that Ruben (green) will win. Investigate various options for this.

Another endgame situation. Ruurd (white) and Ruben (green) both have one block left. It's Ruben's turn. Ruurd already appears to have lost, but appearances can be deceiving.

c. What would you advise Ruben (green) to do: Try to block Ruurd or go for his own score?Substantiate your advice, of course.

Assignment4Scoring

You can see the full Spexxx scoring system in the box at the top right of the board. Make an analysis of this scoring system in which you include the following points, and substantiate everything with mathematical arguments as much as possible:

- is the scoring 'fair'? (And what is 'fair' scoring anyway?)

- what can you say about the data that we have generated together and that you have now received?

- come up with an alternative scoring system

Final assignment and presentation

You can also play Spexxx on a reduced board: the purple and red fields are excluded and instead of throwing four times with five dice you throw three times with four dice.

Design an appropriate scoring system for this alternative game. Use your results from assignment 4. If necessary, also adjust the game rules and/or the game board. Substantiate your scoring system and also use experiences from playing with that system.

Your version may be played during the presentation…

Gold

Objective: after four throws you have five of the same number of eyes.
Strategy: set aside the most common number of eyes from the first throwand continue playing with that number. If there are no two or more of the same number, throwall five dice again.
Do this 50 times and record the number of times you had five equal numbers.

Silver

Objective: after four throws you have a series of five consecutive numbers of eyes.

Strategy: leave the 1 and the 6 as long as possible so as not to fix the sequence immediately.
Do this 50 times and record the number of times you had five consecutive numbers of eyes.

Bronze

Objective: after four throws you have a series of four consecutive numbers of eyes.

Strategy: the 3 and the 4 are in all three possibilities, so take them out first.

Do this 50 times and record the number of times you had four consecutive numbers of eyes.

Red

Objective: after four throws you have four of the same number of pips.
Strategy: set aside the most common number of eyes from the first throw and continue playing with that number. If there are no two or more of the same number, throw all five dice again.
Do this 50 times and record the number of times you had four equal numbers of eyes.

Purple

Objective: after four throws you have three *and* two of the same number of eyes.
Strategy: set aside the most common number of eyes from the first throw and continue playing with that number. If no two or three are the same numbers, throw all five dice again.
Do this 50 times and record the number of times you had three and two equal numbers.

Blue

Objective: after four throws you have three of the same number of eyes.
Strategy: set aside the most common number of eyes from the first throw and continue playing with that number. If there are no two or more of the same number, throwall five dice again.
Do this 50 times and record the number of times you had three equal numbers of eyes.

Green

Objective: after four throws you have two of the same number of eyes.
Strategy: If there are no two or more of the same number, throw all five dice again.
Do this 50 times and record the number of times you had two equal numbers of eyes.